— : — Kinematics of Machines — : —

CH - 1 **Introduction and kinematic chains and Inversion** 20ME43

Introduction :-

* Defn of kinematics :- It is a branch of Mechanics deals with the Study of Relative Motion b/n the parts of M/c without considering Mass of the body & forces acting on it.

⇒ where as considering Mass of the body & forces acting on it is called Dynamics.

※ Definitions and Simple Mechanism :-

※※※ * **Kinematic link (or) Element :-**

It is a part of Machine which has relative motion with respect to another Machine part. It may contain more than one part.

Cylinder Fixed

[* 2 links are joined at same connection → Binary.
⇒ Frame.
* 3 links are joined at same connection. IIIly 4 links. ↳ qualetnary
Ternary.]

In the cylinder (fixed) considering more than one part such as (p) piston, connecting rod (c), and cross-head (C-H).

links can be classified into Binary, ternary and qualetnary depending upon their Ends on which it revolute

Binary link Ternary link Qualetnary link.

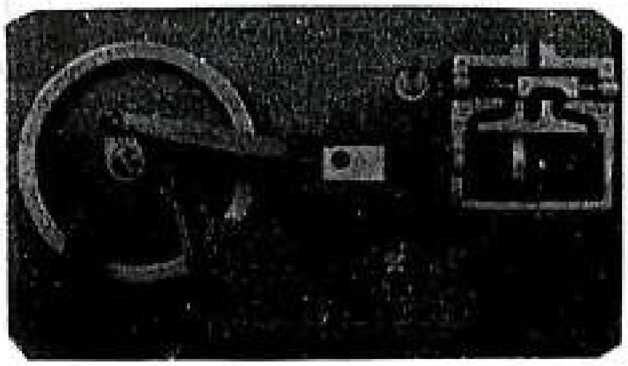
link need not be rigid body but must be resistant body. i.e it must be capable of transmitting required force with negligible deformation.

[Rigid body :- A body is said to be rigid if under action of forces it does not suffer any distortion]

R.S.V.

* Kinematic pair:-

It is a combination of two kinematic links such that their relative motion completely constrained [Constraint – Motion within locus]



Kinematic pair (or) simply pair is a joint of 2 links. From fig the link 2 rotates relative to the link 1 & constitute revolute (or) turning pair. ||ly link 2, 3 & 3, 4 constitute turning pair.

+ Kinematic pairs can be classified a/c to
  a. Nature of contact.
  b. Nature of Mechanical constrains.
  c. Nature of relative motion.

a. Nature of contact:-

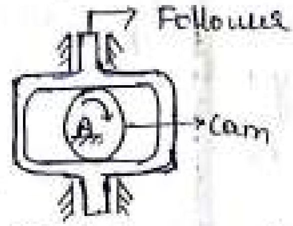
i) lower pair:- A pair of links having surface (or) area contact between the members.
   Eg:- Nut turning on a screw, shaft rotating in bearing &c.

ii) Higher pair:- when a pair has a point (or) line contact b/n the links.
   Eg:- wheel rolling on surface, Ball & Roller bearing etc.

b. Nature of Mechanical constraint :-

..*.

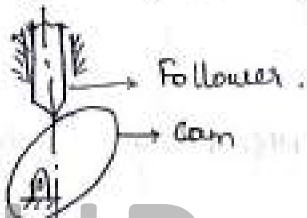
i) closed pair :- when the elements of a pair are held together Mechanically it is known as closed pair.



CAM & Follower pair

The contact between two pairs can be broken only by destruction of atleast one of the Members.

ii) Unclosed pair :- when two links of a pair are in contact either due to force of gravity (or) due to some spring action.

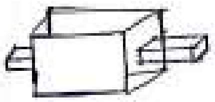


In this, the links are not held together Mechanically, so the contact between two can be broken easily.

c. Nature of Relative Motion :- [constrained Motion]

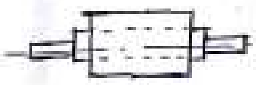
..*.

i) Sliding pair :- If two links have a sliding motion relative to each other, they form sliding pair.



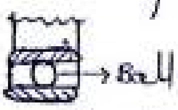
Rectangular rod in Rectangular hole in prism is sliding pair.

ii) Turning pair :- when one link has a turning (or) revolving motion relative to other, they form Turning [Revolving] pair.



A circular shaft revolving inside a bearing is turning pair.

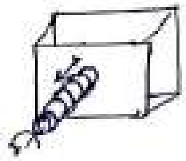
iii) Rolling pair :- when links of a pair having rolling motion relative to each other, they form rolling pair.



The ball & shaft constitute one rolling pair whereas ball & bearing constitute second rolling pair.

P.S.V

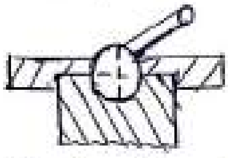
iv) Screw pair (Helical pair) :- If two links have a turning as well as sliding motion between them, form a screw pair.



eg:- Bench vice jaws move with the help of this Mechanism.

The lead screw & the nut of a lathe is a screw pair.

v) Spherical pair :- When one link in the form of a sphere turns inside a fixed link, it is spherical pair.



The ball & socket joint is a spherical pair.

* Motion and its types :-

A change in position is called motion. It is classified as follows:-

* Motion
  i) Absolute Motion ii) Relative Motion.
* Motion
  i) plane motion ii) Rectilinear Motion iii) Helical Motion iv) Spherical Motion
* Motion
  i) Continous motion ii) Reciprocating iii) oscillatory iv) Intermittent
* Motion
  i) uniform ii) variable iii) Simple Harmonic
* Motion
  i) Completely constrained ii) partially constrained iii) Incompletely constrain

* Types of constrained motion in kinematic pair :-

There are 3 types of constrained motion :-

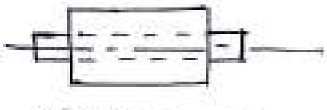
i) Completely constrained ii) Incompletely constrained iii) Successfully constrained.

i) Completely constrained motion :- When motion between two elements of a pair is in definite direction irrespective of direction of force applied.

(a)
Sliding pair   Turning pair.

The above pairs are the Examples of completely constrained motion due to collars at the Ends.

ii) Incompletely constrained motion:- when the motion between two elements of a pair is possible in more than one direction & depends upon the direction of force applied.


Turning pair.

This pair does not have collars. So it possible in more than one direction to direction of force applied.

iii) Successfully constrained Motion:- when motion between two elements of a pair is possible in more than one direction but is made to have motion only in one direction by using External Means.

piston in a cylinder of an I.C. Engine is made to have reciprocating motion & no rotary motion due to constrain of piston pin.

& other Example is Footstep bearing.

* Kinematic chain:-

A Kinematic chain is an assembly of links in which relative motion is completely constrained.

(OR) Kinematic chain is any group of links connected together for the purpose of transmitting force or motion.

(OR) A kinematic chain is a combination of kinematic pairs of Each link forms a part of 2 pairs & relative motion is completely constrained.

* Mechanism:-

By fixing one of the links of a kinematic chain, the arrangement may be used to transmit motion. This arrangement is known as Mechanism.

Mechanism with 4 links is known as "Simple Mechanism".

More than 4 links is known as "Compound Mechanism"

P.S.V

* Linkages:- A linkage is obtained if one of the links of a kinematic chain is fixed to the ground.

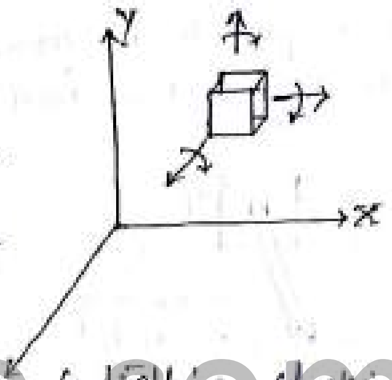
* Structure:- If one of links of redundant chain is fixed, it is known as structure or a locked system.

[Redundant chain]- The chain does not allow any motion of a link relative to other]

Degrees of freedom:-

An unconstrained rigid body moving in space can describe the following independent motion

1. Translation motion along any three mutually $\perp^{r}$ axes x, y & z.
2. Rotational motions about these axes.



Thus a rigid body possess six degrees of freedom.

[Restraints → opp of constrain, To hold back, controlling, checking]

The number of restraints can never be zero [joint is disconnected].

⇒ Thus Degrees of freedom can be defined as number of independent relative motions, both translational & rotational.

So ⟨Degrees of freedom = 6 − No of restraints.⟩

Mobility of Mechanism:-

It defines the number of degrees of freedom.

A Mechanism may consist of a number of pairs belonging to different classes having different number of restraints.

Grubler's criterion:-

To find the number of degrees of freedom for a planar mechanism, we have an equation known as Grubler's Equation & this phenomenon is known as Grubler's criterion.

The Grubler's Equation is given by

$$F = 3(n-1) - 2j_1 - j_2$$

Where $F \rightarrow$ Mobility (or) Number of degrees of freedom

$n \rightarrow$ Number of links including frame.

$j_1 \rightarrow$ Joints with single (one) degrees of freedom.

$j_2 \rightarrow$ Joints with two degrees of freedom.

If $F>0$, results a Mechanism with 'F' degree of freedom.

$F=0$, results in a Statically determinate structure.

$F<0$, results a Statically indeterminate structure.

2005 Jun/July ✱✱✱ Jan/Feb 2004

Degrees of freedom for various joints are given by.

Type of Joint	Nature of Motion	Degrees of freedom
i) Hinges (Revolute)	→ Pure Rolling Motion	→ 1 Degree of freedom.
ii) Slider (prismatic)	→ pure Sliding Motion	→ 1 Dof.
iii) cylindrical, Cam, gear, Ball bearing	→ Rolling & Sliding	→ 2 Dof.
iv) Rolling contact Spherical	→ pure Rolling	→ 1 & 3 Dof

✱✱✱ **Inversion** :- By fixing each link at a time we get as many Mechanisms as number of links, then Each Mechanism is called as an 'Inversion' of the original Kinematic chain.

✱ **Machine** :- It is a combination of resistant bodies with successfully constrained motion which is used to transmit motion to do some useful work.

Eg:- lathe, shaper, Engine's Etc.

✱ **Structure** :- It is an assemblage of a number of resistant bodies having no relative motion b/n them. Structures are meant for taking up loads.

Eg:- Railway bridges, roof trusses Etc.

P.G.V

* Difference between Machine and Mechanism :-

※※※

Machine	Mechanism
1. Machine Modifies Mechanical work.	1. Mechanism transmits Motion
2. A Machine is a development of any Mechanism	2. It is a part of a Machine.
3. Eg:- lathe, Shaper Etc.	3. Eg:- clock work, type-writer Etc.

* Difference between Machine and Structure:-

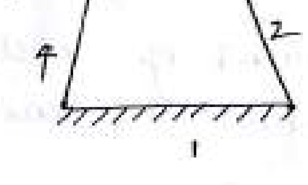
※※※

Machine	Structure
1. Relative Motion Exists	1. No Relative Motion Exists.
2. links are Meant to transmit Motion and force's (both Static and kinetic)	2. Members are Meant for carrying load.
3. Useful Energy is transmitted by it	3. No useful Energy is transmitted by it.
4. Eg:- lathe, Shaper, Engine Etc	4. Eg:- Bridges, Roof trusses Machine frame Etc.

[Not in Syllabus]:-

⇒ For the kinematic linkages shown in Fig 1.18, Calculate the following i) Number of binary links (N_b) ii) Number of ternary links (N_t) iii) Number of quaternary links (N_q) iv) Number of total links (N) v) Number of loops (L) vi) Number of joints (or) pairs (P_1) v) Number of degrees of freedom (F)

i)



W.K.T $F = 3(n-1) - 2j_1 - j_2$

n = Number of links = 4

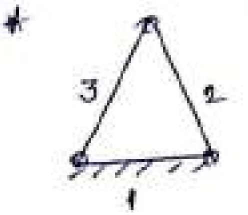
j_1 = Joints with one degree of freedom (lower pair) = 4

j_2 = Joints with two (higher pair) —"— = 0

So $F = 3(4-1) - 2\times4 - 0$.

$F = 9 - 8 - 0$ Hence Mechanism has constrained Motion.

$\langle F = 1 > 0 \rangle$



2) We have $n=3$, $j_1=3$, $j_2=0$

$\therefore F = 3(n-1) - 2j_1 - \cancel{j_2}^{0}$

$= 3(3-1) - 2(3) - 0$

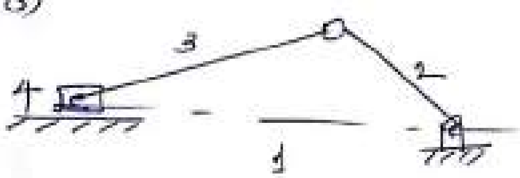
$F = 6-6$

$\langle F=0 \rangle$

Hence this is a statically determinate structure

---

3)



$n=4$, $j_1=4$, $j_2=0$.

$\therefore F = 3(n-1) - 2j_1 - \cancel{j_2}^{0}$

$= 3(4-1) - 2(4)$

$= 9-8$

$\langle F=1 \rangle$

Hence this a Mechanism & has constrained motion.

---

4) To calculate lower pairs (j_1) :-

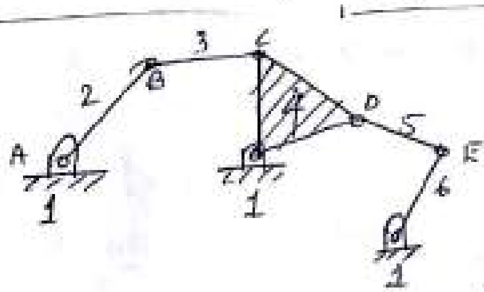
Most of linkages are expected to have one degree of freedom, so the grubler equation becomes.

$1 = 3(n-1) - 2j_1 - \cancel{j_2}^{0}$

So $\langle 2j_1 = 3n-4 \rangle$

---

5)



Here $n=6$, So

$j_1 = \frac{3n-4}{2} = \frac{3(6)-4}{2} = 7$

So, $F = 3(n-1) - 2j_1 - \cancel{j_2}^{0}$

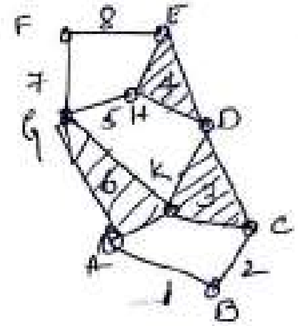
$F = 3(6-1) - 2(7)$

$F = 15-14$

$\langle F=1 \rangle$

$\therefore$ Mechanism has constrained motion.

*6)



Here $n=8$, $j_2=0$

To calculate j_1

we have $2j_1 = 3n-4$

$$j_1 = \frac{3(8)-4}{2} = \frac{20}{2} = 10$$

So $F = 3(n-1) - 2j_1$

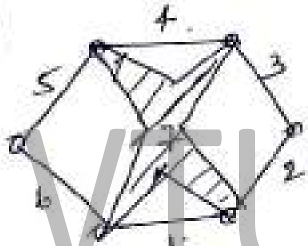
$= 3(8-1) - 2(10)$

$= 21 - 20$

⟨F = 1⟩

Mechanism has constrained motion.

*7)



Here $n=7$, $j_2=0$

To calculate j_1

we have, $2j_1 = 3n-4$

$$j_1 = \frac{3(7)-4}{2} = \frac{21-4}{2} = \frac{17}{2} = 9 \ (8.5)$$

so $F = 3(n-1) - 2j_1$

$= 3(7-1) - 2(9)$

$F = 18 - 18$

⟨F=0⟩

Hence this a statically determinate structure

*8)

$n=14$, $j_2=1$ (Rolling, link 4)

So $j_1 = \frac{3n-4}{2}$ (link 4 is higher pair)

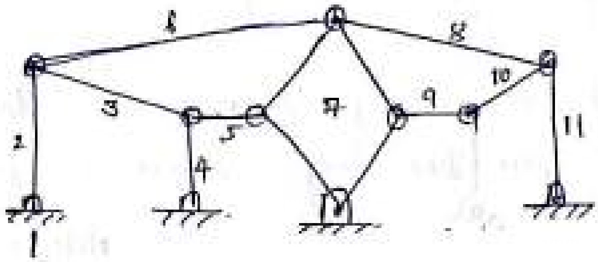
$$j_1 = \frac{3(13)-4}{2} = \frac{39-4}{2} = 18 \ (17.5)$$

So $F = 3(n-1) - 2j_1 - j_2$

$= 3(14-1) - 2\times18 - 1$

⟨∴ F = 2⟩

*** Determine the degrees of freedom of the linkage as shown in fig (July/Aug 2005)



* Degrees of freedom $F = 3(n-1) - 2j$

Here $n = 11$, [~~j = (n+L-1) = 11+5-1 = 15~~] $2j_1 = 3n - 4 \Rightarrow j_1 = \frac{3(11)-4}{2}$

$j_1 = \frac{33-4}{2} = \frac{29}{2} = 14.5$
$= 15$

So $F = 3(11-1) - 2 \times 15 = \underline{0}$

Hence this is a structure.

---

* Prove using Grubler's criterion that for achieving constrained motion the minimum number of links in a mechanism = 4.

* Note :- A binary link consist of two elements.
A ternary link will have 3 elements.
A quaternary link will have 4 elements & so on.

Solution :- W.K.T $F = 3(n-1) - 2j_1 - 2j_2$

If we consider simple mechanism in which $j_2 = 0$ (Number of joint with 2 degrees of freedom.

$\therefore F = 3(n-1) - 2j_1$

j_1 = No of simple hinges, n = Total no of links.

which can be written as

$n = n_2 + n_3 + n_4 + \ldots\ldots$ $n_2 \rightarrow$ Binary link
$n_3 \rightarrow$ ternary link.
$n_4 \rightarrow$ No of qualitary link.

The total no of elements (l) in mechanism is given by

$l = 2j_1 = 2n_2 + 3n_3 + 4n_4 + 5n_5$

substituting the value of n & value of $2j_1$ in Grubler's Equation we get $1 = 3(n_2 + n_3 + n_4 + \ldots -1) - 2n_2 - 3n_3 - 4n_4 - 5n_5$

$\langle n_2 = 4 + \underline{n_4 + 2n_5} + \ldots\ldots \rangle$

P.S.V

As n_4 & n_5 are positive integers, the smallest possible value n_2 is 4.

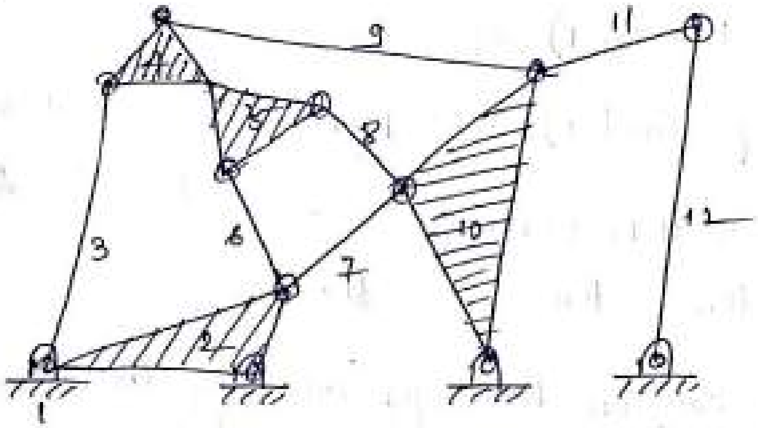
Hence minimum value of binary link is four.

---

* For kinematic linkages shown in fig, the number of binary links, ternary links, other links, total links, loops, joints (or) pairs & DOF.

June/July 2004

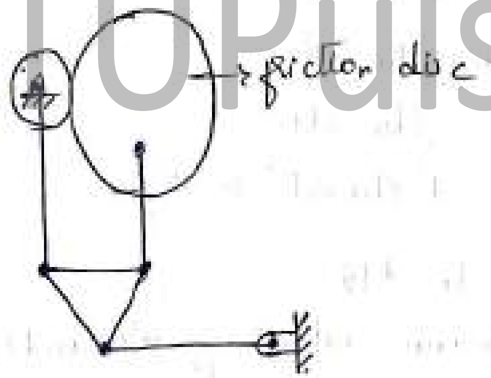
Assignment.



---

* Find DOF.

Aug 2001

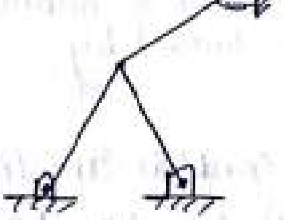


---

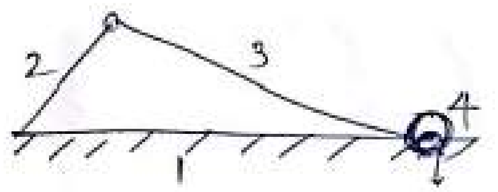
*

*

*



* 9)



Here $n=4,\ j_2=1,\ j_1=3$

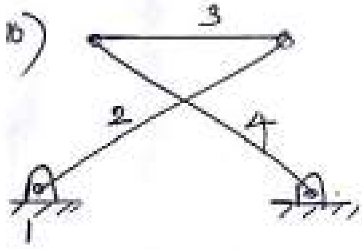
So $F = 3(4-1) - 2\times 3 - 1$

$F = 9-6-1$

$\langle F=2 \rangle$ Hence Mechanism has constrained Motion.

---

* 10)



Here $n=4,\ j_1=4,\ j_2=0$

So $F = 3(4-1) - 2\times 4 - 0$

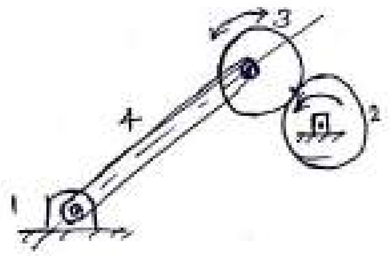
$F = 9-8$

$\langle F=1 \rangle$

Hence this is Mechanism & has constrained Motion.

---

* 11)



Here we have 3 possible cases.

i) Follower will have rolling & sliding. Here we have one higher & 3 Revolute pair.

So $j_1=3,\ j_2=1,\ n=4$

So $F = 3(n-1) - 2j_1 - j_2$

$= 3(4-1) - 2(3) - 1$

$= 9-6-1 = 2$ so $\langle F=2 \rangle$

ii) If link 2 & 3 constitute one line (By welding (d) by some other means)

$n=3,\ j_1=2,\ j_2=1$

$F = 3(3-1) - 2\times 2 - 1 = 1$ $\langle F=1 \rangle$ III^{ly}

iii) cam & follower is pure Rolling

$n=4,\ j_1=4,\ j_2=0$

$F = 3(4-1) - 2\times 4 - 0$

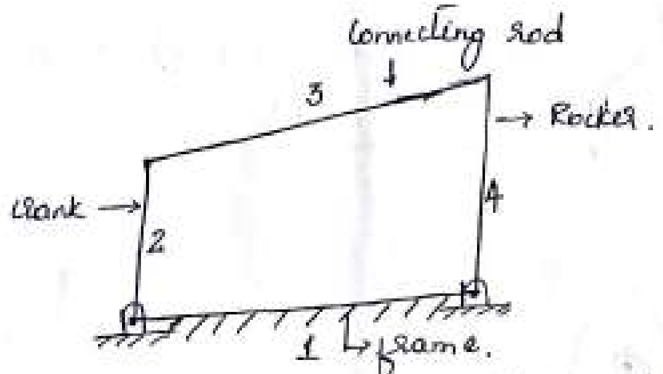
$\langle F=1 \rangle$

* Types of kinematic chain:-

There are 3 types of kinematic chain are there. they are:-

i) Four bar chain.
ii) Slider crank chain.
iii) Double slider crank chain.

i)* Four bar chain (or) Quadric cycle chain:-



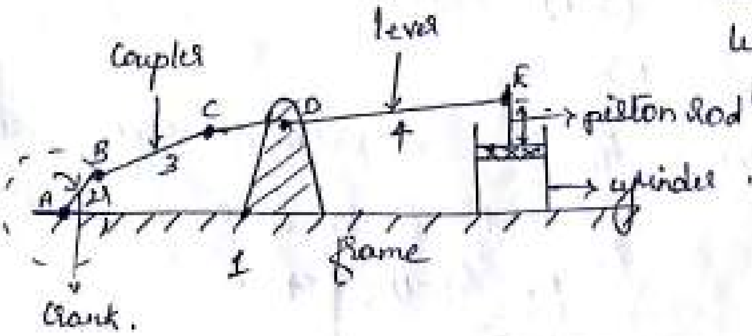
The chain has four links & it is closed cycle frame hence it is also called quadric cycle chain.
In the above figure all four pair will be turning pair (Revolute pair)

→ Inversion of four bar chain Mechanism:-
The important inversions of a four bar chain are the following:-

* Beam Engine (or) Crank and lever Mechanism.
* Coupled wheel of a locomotive (or) Double crank Mechanism.
* Watts straight line Mechanism (or) double lever Mechanism.

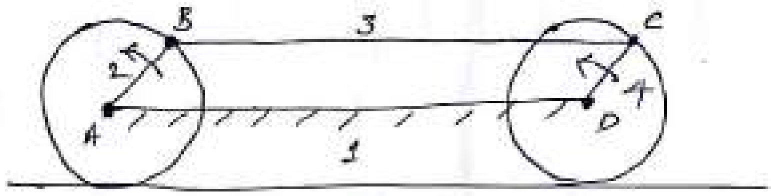
→ Beam Engine:-

link 1 → fixed
link 2 → AB
link 3 → BC
link 4 → DE



The beam Engine Mechanism is shown in above fig. It consists of four links. when crank AB rotates above the fixed centre A, the lever oscillates about a fixed centre D. The End 'E' of the lever CDE is connected to piston rod which moves piston up & down in the cylinder. This is called crank & lever Mechanism.

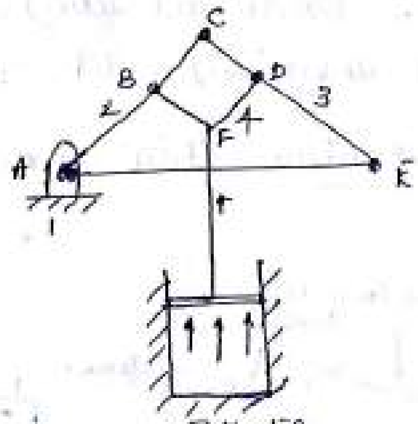
⇒ Coupled wheel of a locomotive (or) Double crank Mechanism:-



link 1 → link AD
link 2 → link AB
link 3 → link BC
link 4 → link CD

In this Mechanism, as shown in fig, the links AB and CD are of equal length and act as cranks. These cranks are connected to the respective wheels. The link BC acts as the connecting rod. The link AD is fixed to maintain constant distance b/n the wheels. This Mechanism is used to transmit rotary motion from one wheel to other. This is called double crank Mechanism.

⇒ Watts Indicator Mechanism:- [Watts straight line Mechanism]



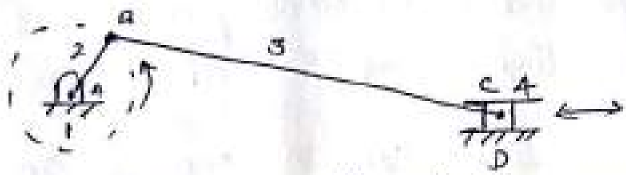
Indicator

link 1 → fixed A.
link 2 → link AC
link 3 → link CE
link 4 → link BFD.

This Mechanism is shown in fig. It consists of four links. The displacement of link BFD is directly proportional to the pressure in the indicator cylinder. the point E on link CE traces out an approximate straight line. It is also called double lever Mechanism.

P.S.V

ii) * Slider Crank Chain :- [Single]



Link 1 → fixed.
Link 2 → AB
Link 3 → BC
Link 4 → prismatic pair CD.

The Mechanism is shown in the above figure. It consists of three turning pair and one sliding pair. link 1 corresponds to frame which is fixed. link 2 is the crank & link 3 the connecting rod. Link 4 is the slider, used to convert rotary motion into reciprocating motion.

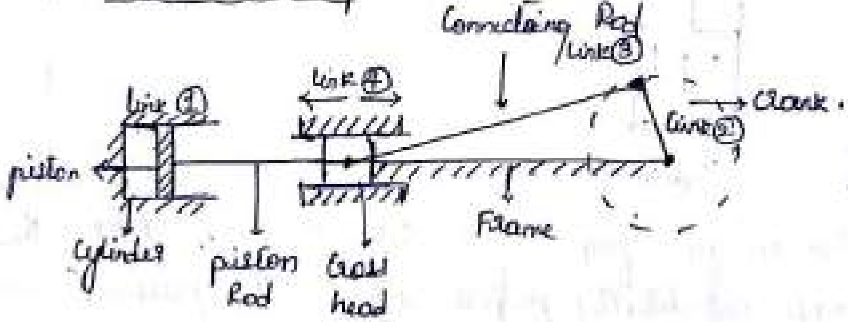
* Inversion of Single Slider Crank chain :-

There are 4 inversion's in a single slider crank chain Mechanism, they are

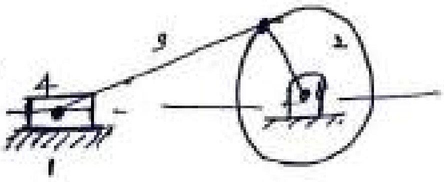
i) Reciprocating Engine Mechanism (First inversion) [cylinder & frame] [Fixed]
ii) a. Oscillating cylinder Engine Mechanism (Second inversion) [connecting Rod]
b. Crank and slotted lever Mechanism (Second inversion)
iii) a. whitworth quick return motion Mechanism (Third inversion) [crank]
b. Rotary Engine Mechanism (Third inversion) [crank]
IV) a. Bull Engine Mechanism (Fourth inversion) [slider & piston]
b. Hand pump (Fourth inversion) [slider & piston]

* Reciprocating Engine Mechanism :- (First inversion)

* Actual set-up:-



* line diagram:-



In the first inversion, the link 1 ie cylinder & frame is fixed. link 2 is crank, link 3 is connecting rod, & link 4 is crosshead. As crank rotates the cross head reciprocates & thus piston reciprocates in cylinder. This is Reciprocating Engine Mechanism.

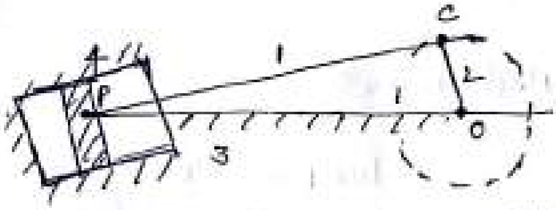
This Mechanism is used in Steam Engines, pumps, compressors, I.C. Engines etc. ⑧

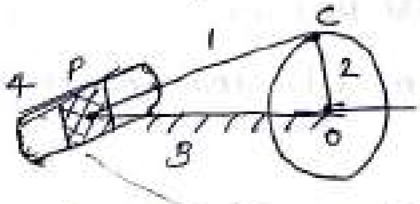
* Second Inversion:-

a. Oscillating cylinder Engine Mechanism:-

Actual Setup:-

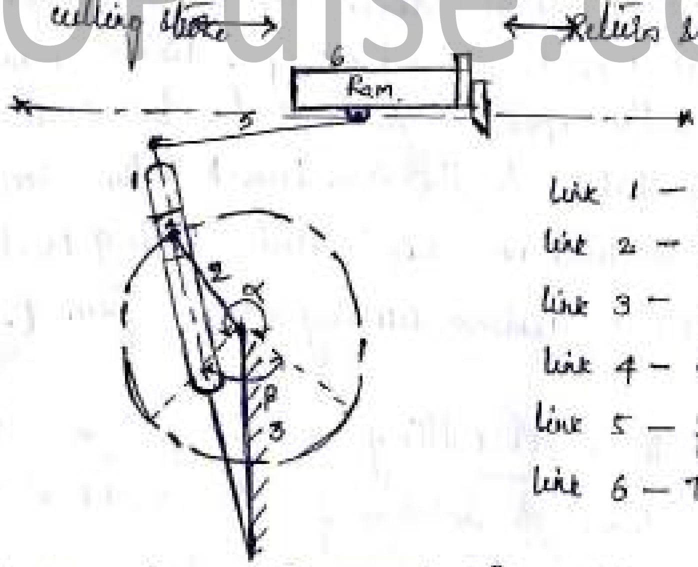
line diagram





Second inversion is obtained by fixing the connecting Rod (or) link 3. Here connecting Rod PO is fixed. When Crank OC (link 2) rotates the piston attached to piston rod (link 1) reciprocates & cylinder (link 4) oscillate about p.

b. Crank and Slotted Lever Mechanism :-



link 1 — Slotted
link 2 — Crank
link 3 — fixed link.
link 4 — Slider.
link 5 — floating link.
link 6 — Tool holder.

In this Mechanism link 3 is fixed. The Slider (link 4) reciprocates in slotted lever (link 1) & crank (link 2) rotates. link 5 connects the link 1 to the ram (link 6). The ram with cutting tool reciprocates ⊥er to fixed link 3. The ram with tool reverse its direction of motion when link 2 is ⊥er to link 1. Thus cutting stroke is executed during rotation of crank through angle 'α' & return stroke is executed through angle 'β' (or) "360 − α"

P.T.O

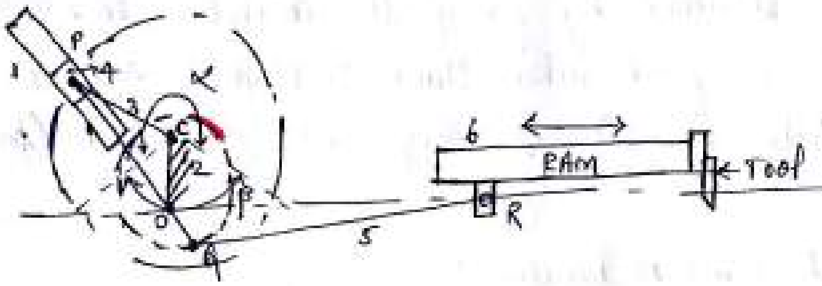
Therefore we get

$$\frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

+ This Mechanism is used in Shaping Machine, Slotting Machine & in Rotary Engine.

+ Third Mechanism :-

a. Whitworth quick return motion Mechanism :-



link 1 → slotted lever
link 2 → fixed link.
link 3 → driving link.
link 4 → slider.
link 5 → floating link.
link 6 → Tool holder.

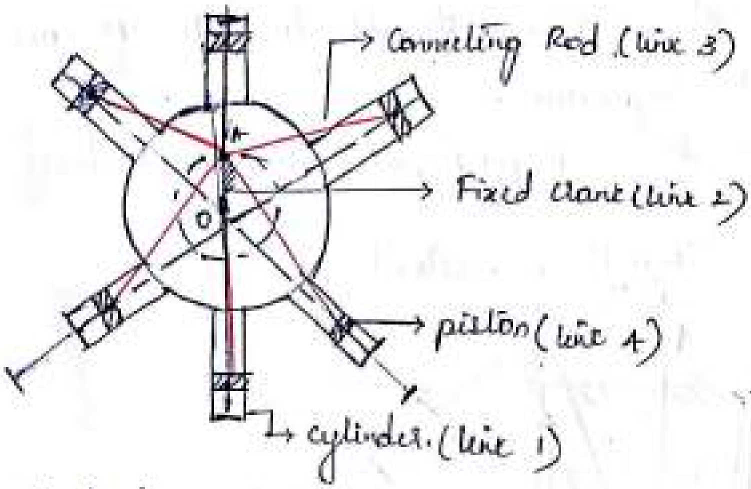
Third inversion is obtained by fixing the crank i.e link 2, the crank OC is fixed & OQ rotates about O. The slider slides in the slotted link & generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to ram. The rotary motion of P is taken to the arm ram R which reciprocates.

This Mechanism is used in Shapers and slotting Machines.

The angle obtained during cutting stroke from P in counter clockwise direction.

$$\therefore \frac{\text{Time for cutting}}{\text{Time for return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

b. Rotary internal combustion Engine (Gnome Engine) :- (Third inversion)

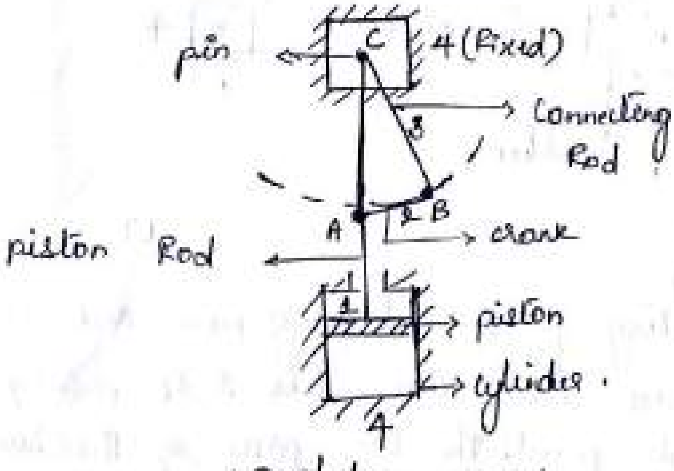


Rotary Engine Mechanism (or) Gnome Engine is another application of third inversion. It is a rotary cylinder V-type internal combustion Engine used as an aero-Engine. But now Gnome Engine has been replaced by Gas turbine. The crank OA is fixed and all connecting rods from the piston are connected to A. In this Mechanism when pistons reciprocate in the cylinder.

The whole assembly of cylinder pistons & connecting rods rotate about the axis o, where entire Mechanical power developed is obtained in the form of rotation of the crank shaft.

Fourth inversion :-

a. Bull Engine Mechanism (or) pendulum pump :- (4th inversion)

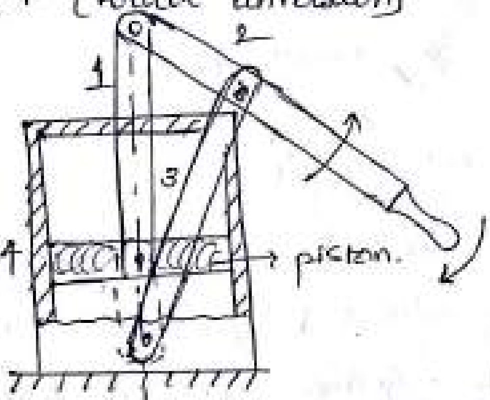


: pendulum pump :-

link 1: cylinder.
link 2: crank.
link 3: connecting rod.
link 4: Fixed link.

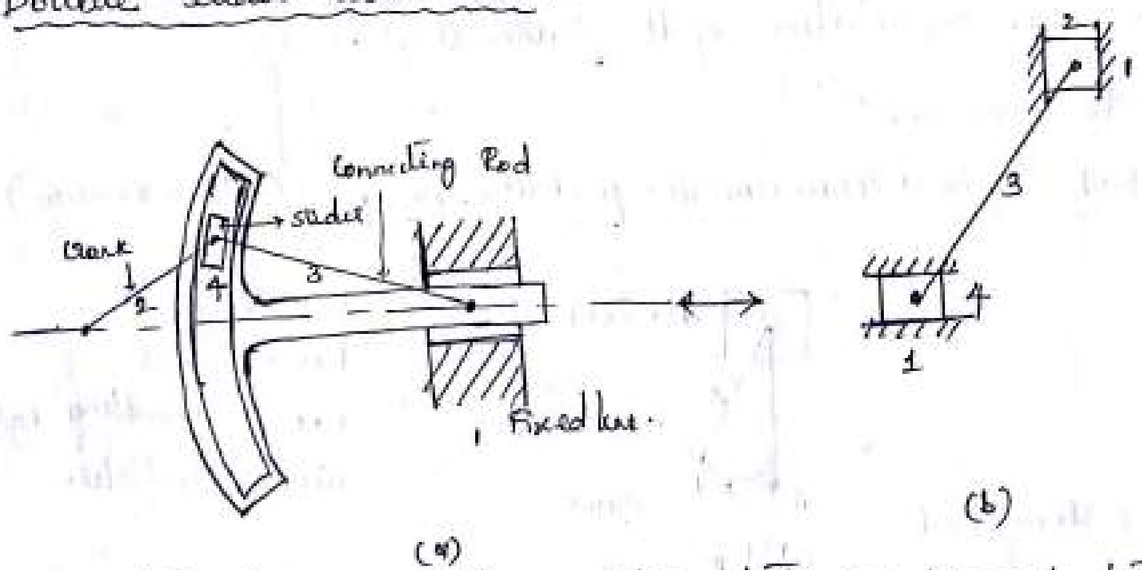
Fourth inversion is obtained by fixing the slider (or) piston i.e link 4. link 2 rotates about A, (link BC) i.e link 3 will oscillate & link 1 will reciprocate along vertical straight line. This Mechanism doesn't have much practical importance.
It's application is to supply feed water to boiler.

b. Hand pump :- [Fourth inversion]



Hence it is fourth inversion the slider ie link 4 is fixed. Here also the slotted link shape is given to slider & vice versa. Here link 1 reciprocate along vertical straight line, at the same time link 2 will rotate & link 3 will oscillate.

* Double slider-crank chain :-



This kinematic chain consisting of two turning pairs and two sliding pairs is called double slider - crank chain. link 3 & 4 reciprocate. link 2 rotates & link 1 is fixed. The two pair of the same kind are adjacent.

P.S.V

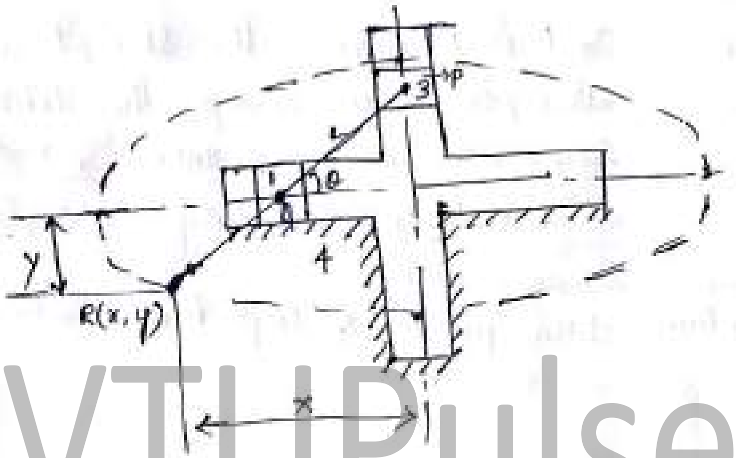
* Inversion of Double Slider-Crank chain:-

The three Important inversions of double slider Crank chain are:-

1) Elliptical trammel.
2) Scotch yoke mechanism.
3) Oldham's coupling.

*i) Elliptical trammel:-

This is a device (or) instrument for drawing Ellipses. Which we have 2 grooves are cut at right angles in a plate which is fixed.



Here we have x & y values respectively.

$x = PR\cos\theta$. (or) $\frac{x}{PR} = \cos\theta$. —— ①

$y = QR\sin\theta$. (or) $\frac{y}{QR} = \sin\theta$. —— ②

Squaring and adding ① & ②, we get.

$$\frac{x^2}{(PR)^2} + \frac{y^2}{(QR)^2} = \cos^2\theta + \sin^2\theta$$

$$\left\langle \frac{x^2}{(PR)^2} + \frac{y^2}{(QR)^2} = 1 \right\rangle$$

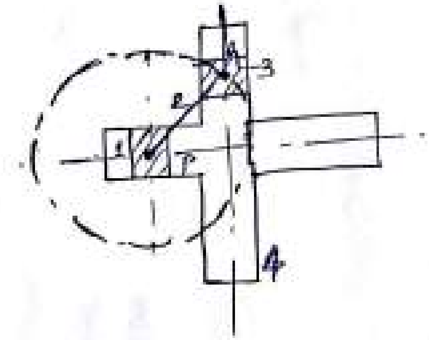
This is the Equation of Ellipse, hence the instrument traces an Ellipse.

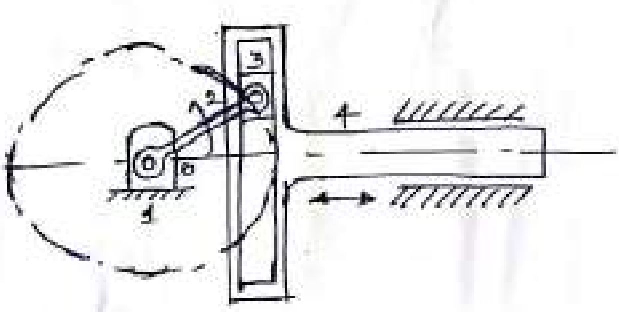
* If tracing point 'R' is assumed to be the Mid-point of PQ. then the path traced by R is circle, so then PR = QR

$$\left\langle \therefore \frac{x^2}{PR^2} + \frac{y^2}{PR^2} = 1 \right\rangle$$ when O^{le} with PR Radius.

R.S.V

2. Scotch Yoke Mechanism:-



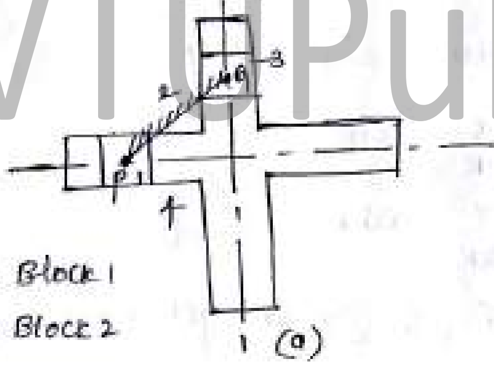


link 1 - slider, fixed link
link 2 — connecting Rod.
link 3 — slider.
link 4 — slotted frame

In this Mechanism, the slider 'P' is fixed. When PQ Rotates about P, the slider Q reciprocates in vertical slot. This Mechanism is used to convert Rotary to reciprocating motion & this Mechanism is shown in fig.

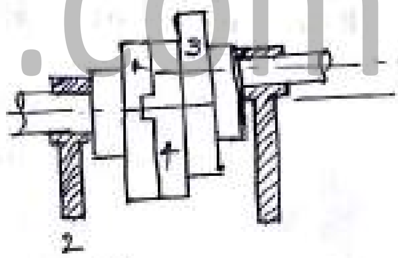
Application:- steam pumps & to produce vibrations.

3) Oldham's coupling:-



P[1] → Block 1
Q[3] → Block 2

1 (a)



Oldham's coupling

- In these Inversion the link connecting the two blocks P & Q is fixed. If one block is turning through an angle, the frame & other block will also turn through the same angle as shown in fig(a)
- The application of this Inversion is Oldham's coupling as in fig(b). Here the two parallel shaft is connected through 2 flanges [1 & 3] & intermediate disc [4], link 2 is fixed.
- When flange 1 turns, the Intermediate disc 4 must turn through same angle & whatever angle 4 turns through, the flange 3 must turn through same angle. Hence 1, 4 & 3 must have same angular velocity.

The Max sliding speed of each tongue along slot is given by $\langle v = x\omega \rangle$, $\omega \rightarrow$ angular velocity rad/sec, v = linear velocity m/sec.

* Mechanism :- By fixing one of the links of a kinematic chain, the arrangement may be used to transmit Motion. This arrangement is known as Mechanism.

The Main Mechanism is Quick Return Motion Mechanism

* Quick Return Motion Mechanism :-

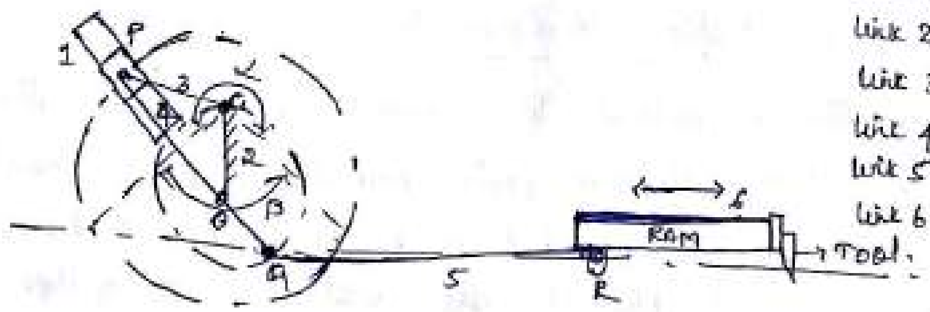
Many times Mechanisms are designed to perform repetitive operations.

⇒ Working Stroke :- During continuous (or) repetitive operations for a certain period the Mechanism will be underload known as working stroke (or) cutting stroke.

⇒ Return Stroke :- Remaining period of the Mechanism is called as Return Stroke, where this Mechanism returns to repeat the operation without load.

The Various quick return Motion Mechanisms are :-
i) Whitworth quick return Mechanism.
ii) Drag link Mechanism.
iii) Crank and Slotted lever Mechanism.

i) Whitworth Quick Return Mechanism :-



Link 1 → Slotted lever.
Link 2 → fixed link.
Link 3 → driving link
Link 4 → slider.
Link 5 → floating link.
Link 6 → Tool holder.

This Mechanism is obtained by fixing the crank i.e link 2, The crank OC is fixed & OA rotate about O. The slider slides on the slotted link & generates a circle of radius CP. Link 5 connects the Extension OA provided on opp side of the link 1 to ram. The rotary motion of P is taken the ram 'R' which reciprocates.

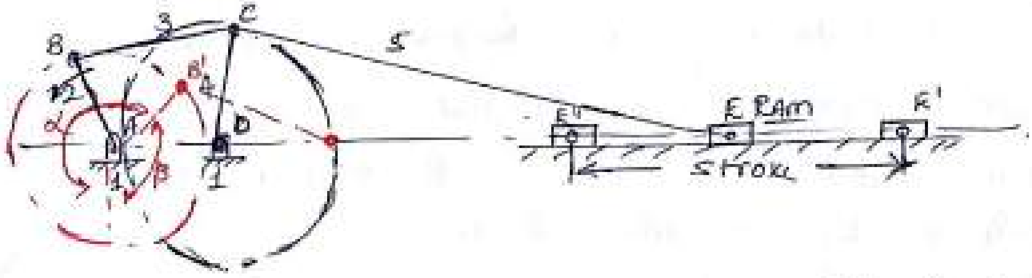
This Mechanism is used in shaper & slotting M/C

P.S.V

The angle obtained during cutting stroke in counter clockwise direction.

$$\therefore \frac{\text{Time for cutting}}{\text{Time for return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$$

ii) Drag link Mechanism :-



In this Mechanism the crank AB rotates at uniform speed, the crank CD will rotate at non uniform speed. This rotation of link CD is transformed to Quick return reciprocatory motion of ram E by link CE.

Here time of working stroke $= \frac{\alpha}{\beta} = \frac{\alpha}{360-\alpha}$.

iii) Crank & slotted lever Mechanism :-

Refer 1st chapter, page No "8".

* straight line Motion Mechanism :-

The easiest Method to generate a straight line Motion is by using sliding pair, but in precision Machines sliding pair are not preferred because of wear & tear. Hence in such cases different Method's are used. They are called straight line Motion Mechanism.

There are two types, they are:-

1. Exact straight line Motion Mechanism.
2. Approximate straight line motion Mechanism.

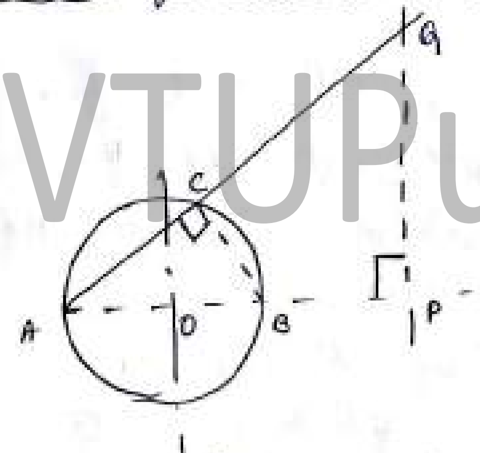
Again Exact straight line Motion Mechanism is classified into 3 types.

1) peaucellier mechanism.
2) Hart Mechanism.
3) scott - Russel Mechanism.

& Approximate straight line motion Mechanism is classified into 4 types.

1) Watt Mechanism.
2) grasshopper Mechanism.
3) Roberts Mechanism.
4) Tchebichaff's Mechanism

* Condition for Exact straight line Motion Mechanism:-



The principle adopted for a Exact straight line motion is shown in fig.
Consider a circle & A be a point on the circumference of circle of diameter AB. let Ac be any chord & Q is a point on Ac produced. where 'AQ' is constant. Then path of Q will be a straight line & its is ⊥er to diameter of AB. This may be proved as follows:-

Draw Ap ⊥er to OB produced, join CB. & sles ACB & ApQ are similar.

$$\text{So}\quad \frac{AC}{Ap} = \frac{AB}{AQ}$$

$$AC \cdot AQ = AB \cdot Ap.$$

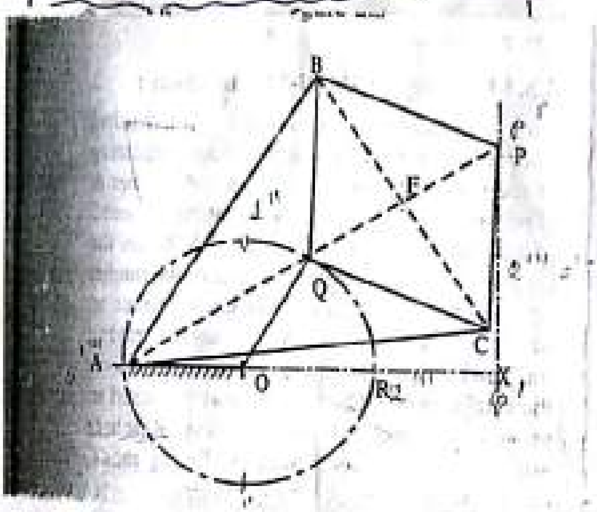
$$Ap = \frac{AC \cdot AQ}{AB}$$

AB is constant as it is the diameter of cle. ∴ if the product AC.AQ is const then Ap will also be const. Hence the point 'Q' moves along straight path Qp which is ⊥er to Ap.

One among them is peaucellier mechanism.

4) Exact straight line Mechanism:-

a. Peaucellier Mechanism:-



Here the line OA is fixed & connected to the link OQ. The pin Q will move along circumference of a circle by means of the link OQ.

The link OQ & OA line are equal in length. On Q 4 bar chain Mechanism is present & has 4 links QC, CP, PB & BQ. Here link AB=AC

To prove this Mechanism is having straight line Motion, we by proving the product AQ.Ap remains constant.

Join BC to bisect PQ at F. Then R.angle △ls AFB, BFP, we have $AB^2 = (AF)^2 + (FB)^2$ — (pythagorus theorm) — ①

$|||^{ly}$ $BP^2 = (BF)^2 + (FP)^2$ — ②

① - ② we get $AB^2 - BP^2 = (AF)^2 + \cancel{(FB)^2} - \cancel{(BF)^2} + (FP)^2$

So $AB^2 - BP^2 = AF^2 - FP^2$

This can write as $= (AF - FP) \times (AF + FP)$

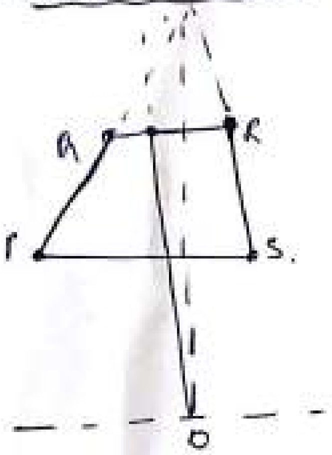
$AB^2 - BP^2 = AQ \cdot AP$

Since AB & BP are links of const length, the product AQ.AP is constant. Therefore point 'p' traces straight path normal to AR.

* Approximate straight line Motion Mechanism :-

a) Roberts Mechanism :- This is also a 4 bar chain. The links PQ & RS of equal length with different inclination angle.

The link 'O' is rigidly attached to line QR at right angle. The path ~~path~~ of 'O' is clearly approximately horizontal in this Mechanism.



* Intermittent Motion Mechanism :-

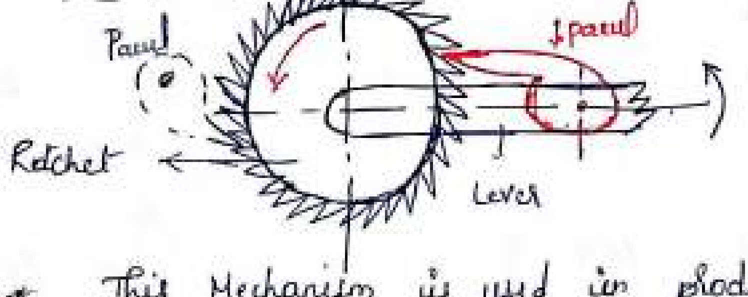
Intermittent Motion Means that the Motion is not continous but it is ceased at definite interval. There are Many instance where it is necessary to convert continous Motion into intermitten Motion.

Eg:- Gear Motion etc.

We have 2 types of Mechanism, they are:-

* a. Ratchet & Paul Mechanism.
* b. Geneva Mechanism

a. Ratchet & paul Mechanism :-

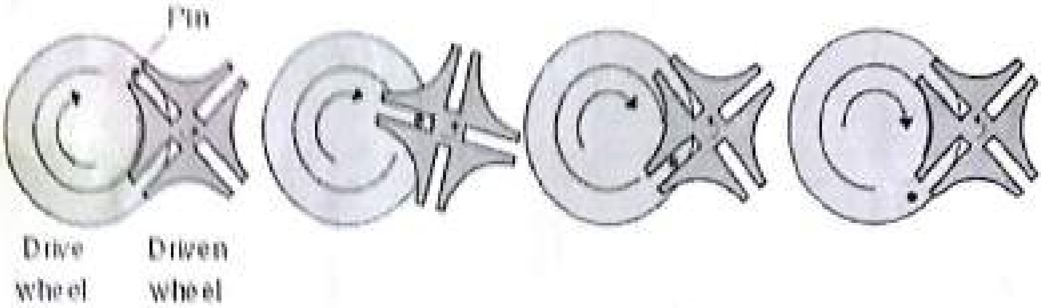


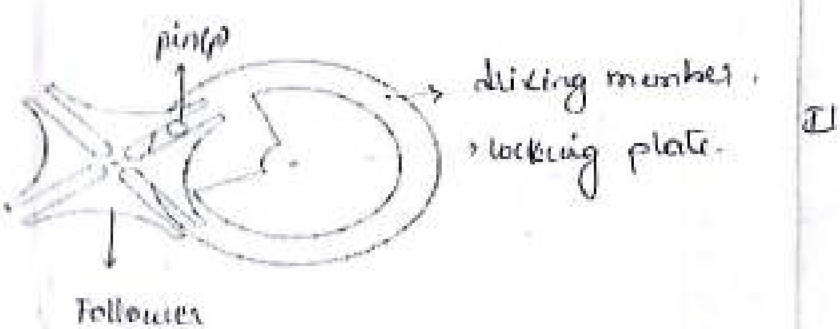
* This Mechanism is used in producing intermittent rotary Motion from an oscillating (or) reciprocating Motion Member.
* Mainly this Mechanism consists of Ratchet wheel, paul, lever. when the lever 4 carrying the paul 3 is ~~lowered~~ raised, the ratchet wheel rotate in counter clockwise direction. One More paul is used to prevent the ratchet from reversing.
* This Mechanism are used in feed Mechanism, lifting jacks, watches & counting device.

P.S.V

b. Geneva Mechanism :-

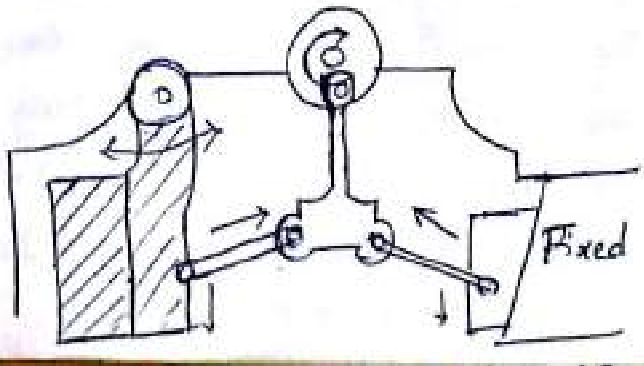
Application :- Movie projectors, Stepper motor, watches &c

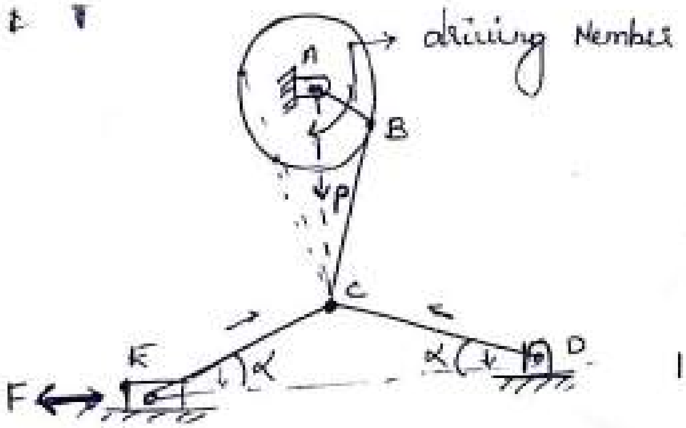




* Geneva Mechanism is an intermittent Motion Mechanism. It consists of driving wheel carrying pin & locking plate. The pin p which Engages in a slot of the follower 'F' as shown in the above figure.
* During one quarter revolution of the driving the pin & follower remain in contact & hence the follower is turned by one quarter turn, the follower remains at rest locked in position by circular arc.
* locking plate is provided to lock the follower when it is not being indexed.

* Toggle Mechanism :-



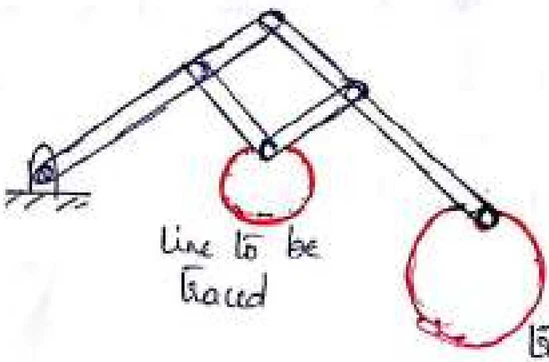


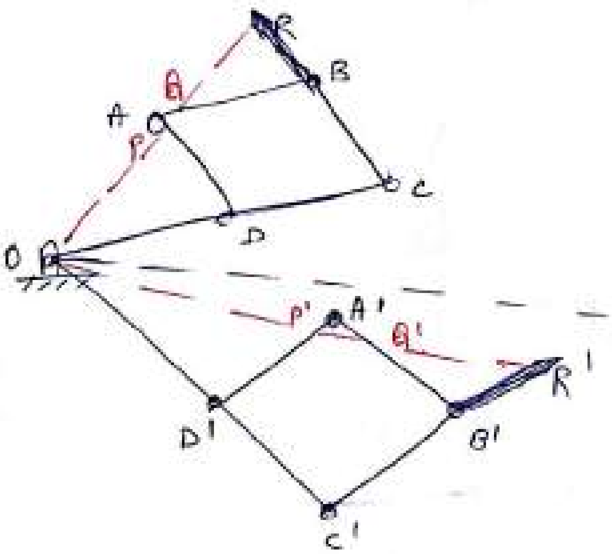
when driving link(AB) rotates, the link BC will inturn rotate. As link BC is interconnected to CE & DC, this both link interact with the link BC. Here link CE slides & CD rotate.
The CE & CD link will gain movement with angle α.
Application:- stone crusher, punch presses, rivetting Machines etc.
By Resolving the force in the above fig.

$$F \tan\alpha = \frac{P}{2}$$

$$\therefore F = \frac{P}{2\tan\alpha}$$

* Pantograph:-





* Pantograph is a four-bar linkage used to produce paths exactly similar to the ones traced out by a point on the linkage. The paths so produced are usually on an enlarged (or) reduced scale & may be straight (or) curved ones.

* Four links of a pantograph are arranged in such a way that a parallelogram ABCD is formed. Thus AB = DC & BC = AD. If some point O in one of the links is made fixed & three other points P, Q & R on other 3 links are located in such a way that OPQR is straight line.

* It can be shown that points P, Q & R always move $\parallel^{el}$ & $\parallel^{el}$ to each other over any path, straight (or) curved. Their motion will be proportional to their distance from fixed point.

* Let O, P, Q & R be on links CD, DA, AB & BC respectively. ABCD is the initial position as shown in figure.

* Let the linkage be moved to another position so that A moves to A', B to B' & so on.

In Δ^{le} ODP & Δ^{le} OCR

O, P & R lie on a straight line & thus OP & OR coincide.

$\angle DOP = \angle COR$

$\angle ODP = \angle OCR$.

$\therefore$ Δ^{les} are similar.

& $\frac{OD}{CC} = \frac{OP}{OR} = \frac{DP}{CR}$

Now, $A'B' = AB = DC = D'C'$

& $B'C' = BC = AD = A'D'$

$\therefore$ $A'B'C'D'$ is again a parallelogram.

In Δs $OD'P'$ & $OC'R'$

$$\frac{OD'}{OC'} = \frac{OD'}{OC'} = \frac{DP'}{CR'} \quad \text{— (1)}$$

$$= \frac{D'P'}{C'R'} \quad [\text{from Eqn (1)}]$$

& $\angle OD'P'$ & $\angle OC'R'$

$|||^{ly}$ O, P' & R' lie on straight line.

$$\frac{OP}{OR} = \frac{OD}{OC}$$

$$= \frac{OD'}{OC'}$$

$$= \frac{OP'}{OR'}$$

$\therefore$ Δ^{s} $OD'P'$ & $OC'R'$ are $|||^{er}$

This shows the initial parallelogram & final $\square$gm which moved & located is equal but enlarged (or) reduced.

* Steering Mechanism :— Steering gear are used to control direction of motion in vehicles.

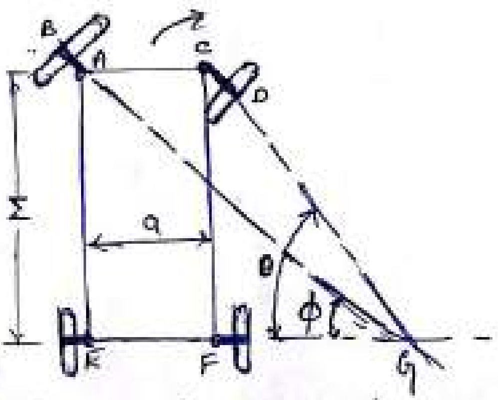
The Mechanism which provides relative motion between the wheels of the vehicle & road surface which is purely rolling.

+ Condition for correct steering in motor car:-

In order to have pure rolling motion b/n road surface & wheels along a curved path, the steering gear must be so designed that the path's of point of contact of each wheel with the ground are concentric circular arcs.

Steering is usually effected by turning the axes of rotation of the two front wheels relative to body of vehicle.

⇒ To satisfy above condition the axle of wheel on the inside of curve must be turned through a larger angle than axis of the wheel on outside of the curve.



ACEF → chassis (body)
AB, CD → axles
θ → angle of axis of wheel inside of curve.
ϕ → angle of axis of wheel outside of curve

When turning to right the axes AB & CD intersect in the common axis EF of the rear wheels at the point G.

Here $AE = CF$ & $AC = EF$

So $AC = EG - FG$ —① $[\because EF = EG - FG]$

so $\tan\theta = \frac{CF}{FG}$ $\left[\because \tan\theta = \frac{opp}{adj}\right]$

$\therefore FG = \frac{CF}{\tan\theta} = CF\cot\theta$ —— ②.

& $\tan\phi = \frac{AE}{EG}$

$\therefore EG = \frac{AE}{\tan\phi} = AE\cot\phi$ —— ③.

Substituting ③ & ② in eqn ①

$$AC = AE\cot\phi - CF\cot\theta$$

W.K.T $AE = CF$

$$\therefore AC = AE[\cot\phi - \cot\theta]$$

$$\left\langle \cot\phi - \cot\theta = \frac{AC}{AE} = \frac{a}{w} \right\rangle$$

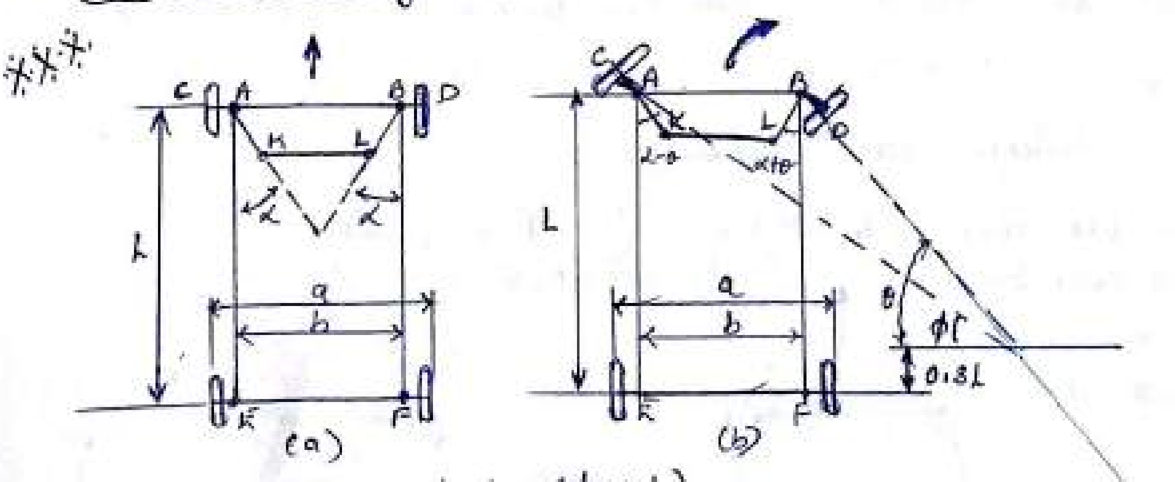
Hence this is the condition for correct steering.

We have two types of steering, Mainly we have

a) Ackermann steering b) Davis steering

2004, 2007, 2011

* Ackermann steering gear Mechanism :-



ABEF → body (chassis)

KL → shorter link [$\parallel^{el}$]

AK, & BL → shorter link [inclined]

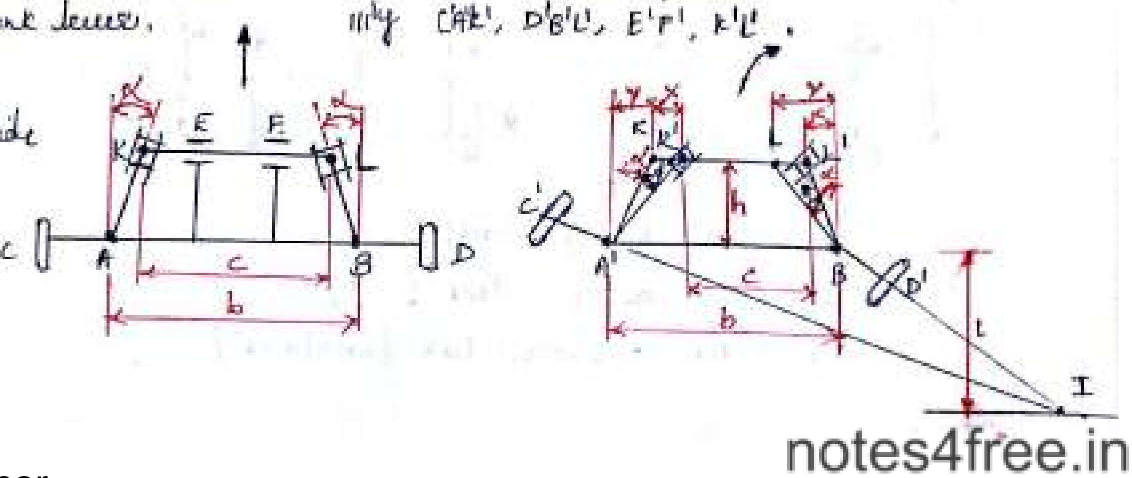
P.S.V

The above figure shows the arrangement of Ackermann steering gear Mechanism. The Mechanism is Made of only turning pairs & is based on 4-bar Mechanism.

* Cross link KL connects two short axles AC & BD of the front wheels through short links AK & BL which form bell crank lever CAK & DBL respectively, ABLK is 4-bar Mechanism.
* Fig (a) shows the vehicle is Moving along straight path & Fig (b) shows the vehicle is Steering Right
* In Fig (b) the short link BL is turned so to Increase 'α', also link LK causes the turn so as to reduce 'α'.
* The fundamental Equation for correct steering is,
  $$\cot\phi - \cot\theta = b/L$$
* By above arrangement it is clear that angle ϕ through which AK turn is less than angle θ through which BL turns & therefore the left front axle turn through a smaller angle than Right front axle.
* For different angle of turn θ, the corresponding value of ϕ are noted. "Approximate value of "b/L" for correct steering should be between "0.4 & 0.5""
* The Intersection of two axis doesnot lie on the axis of rear axle but on a line $\|^{el}$ to rear axle axis at an approximate distance of 0.3 L above it.

* b) Davis Steering Gear Mechanism:

where KL = cross link which slides $\|^{el}$ to AB, K'L' = Final position.
CAK, DBL = Bell Crank levers, $\|^{ly}$ C'A'K', D'B'L', E'F', K'L'.
E, F = bearing
K, L = pins with slide blocks.
AC, BD = stub axles
A, B = pivots.



* Determination of angle 'α':

The Davis steering gear is shown in figure, when automobile is turning right, In this position we have.

$$y = AK \sin\alpha = BL \sin\alpha$$

where $y = \tan(\alpha-\theta)$, $A\sin\alpha = BL\sin\alpha = \frac{y-x}{h}$.

$$\text{So} \quad \tan(\alpha-\theta) = \frac{y-x}{h}$$

$$\frac{\tan\alpha - \tan\theta}{1+\tan\alpha\tan\theta} = \frac{y-x}{h} \qquad \text{But } \tan\alpha = y/h$$

$$\frac{(y/h) - \tan\theta}{1+(y/h)\tan\theta} = \frac{y-x}{h}$$

$$\tan\theta = \frac{xh}{y^2-xy+h^2} \longrightarrow ①$$

$$\text{III}^{ly} \quad \tan(\alpha+\phi) = \frac{y+x}{h}$$

$$\text{So} \quad \tan\phi = \frac{xh}{y^2+xy+h^2} \longrightarrow ②$$

Reciprocal of ② - ①, we get.

$$\cot\phi - \cot\theta = \cancel{y^2}+\cancel{xy}+h^2 - \cancel{y^2}+\cancel{xy} - \frac{h^2}{xh} = b/l$$

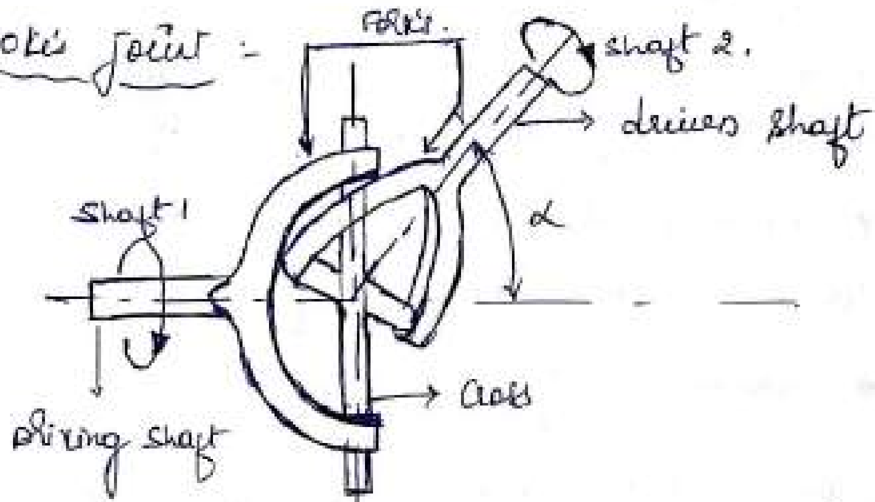
(or) $\frac{2y}{h} = \frac{b}{l}$ [$\because$ h = height b/w & II^el axels, y = Intial position of axels. So h = 4 & y = b.]

$$\text{So} \quad \tan\alpha = \frac{b}{2l}$$

Generally b/l = 0.4 to 0.5 for the Davis steering gear Mechanism

P.S.V

* Hooke's joint :-



* Hooke's joint used to connect 2 non parallel shafts. The can also be ~~they~~ used for shaft with angular misalignment.
* Hence Hooke's joint is a mean of connecting 2 rotating shafts whose axes lie in the same plane & direction making small angle.
* It mainly consists of 2 U-shaped yokes [forks] which are driving & driven members & cross shaped connecting link.
* When shaft 1 rotates the fork of shaft 1 rotates, this fork touches the other fork during rotation, this so this fork rotates inturn shaft 2 rotates.
* There are 2 types of Hooke's joint, they are.
  i) Single Hooke's joint.
  ii) Double —— " ——.
* Application
  i) Transmission of power from gear box to rear axle
  ii) Milling Machine.
  iii) Multiple drilling Machine.

-: Spur Gears :-

* **Gear**: A gear may be defined as any toothed member designed to transmit (or) receive motion from another member by successively engaging teeth.

⇒ **Application**: Used in metal cutting machine tools, automobiles, tractors, hoisting & transporting machinery, rolling mills etc.

* The gears give positive drive & provide many advantages over friction drives like belts, ropes & friction drum.

* The error in tooth meshing may cause undesirable vibration & noise during operation.

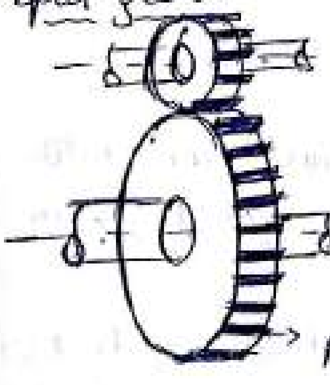
* Smaller gear is called pinion & bigger one is called gear wheel.

* **Types of gears**:-

The gear may be classified as follows:-

i) Spur gear ii) Helical gear iii) Haringbone gear iv) Bevel gear
v) Worm gear vi) Rack and pinion.

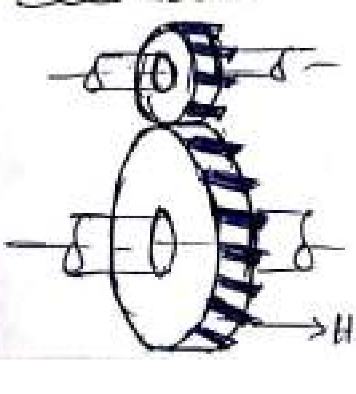
* i) **Spur gear**:-



A spur gear is a cylindrical gear whose tooth traces are straight line & parallel to axis of shaft.

* This gear is used in machine tools, automobile etc.

* **Helical gear**:-

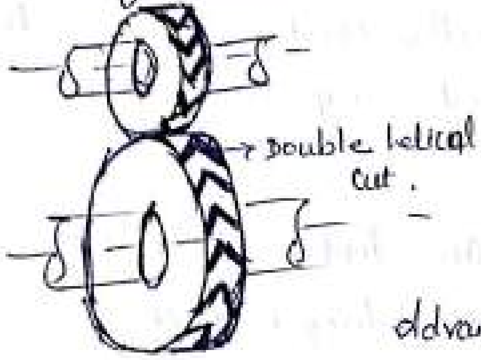


The helical gear is similar to spur gear in which tooth traces helical shape.

* This gear is used to carry heavy load, gives low noise & to have smooth operation,

P.S.V.

iii) Herringbone gear :- [Double helical gear]

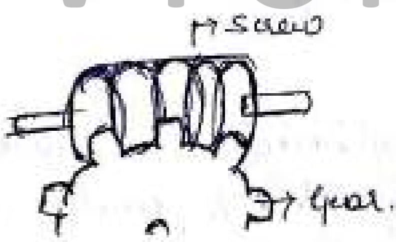


This gear is also called Double helical gear. It is a special type of gear combination of two helical cuts of opposite hand.

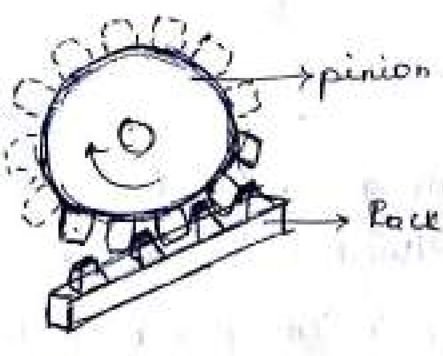
Advantage of transferring power smoothly as multiple gear teeth engage & disengage simultaneously.

iv) Bevel gear :- In this gear the straight teeth is trace over the tapered surface i.e on the frustum of cone.
It is used to connect 2 shafts at any angle & to transmit motion.

v) Worm gear :- This gear has screw and gear wheel. where rotary motion is converted into linear motion.



vi) Rack and pinion :- Rack is a rectangular block with no of teeth as shown in fig & pinion a gear wheel.



where pinion is a driver & Rack is a driven in this mechanism. The function of rack & pino

* Types of gearing :-

There are 2 types of gearing :-

i) Internal gearing :-

In this gearing, the gears mesh internally & they rotate in the same direction.

ii) External gearing :-

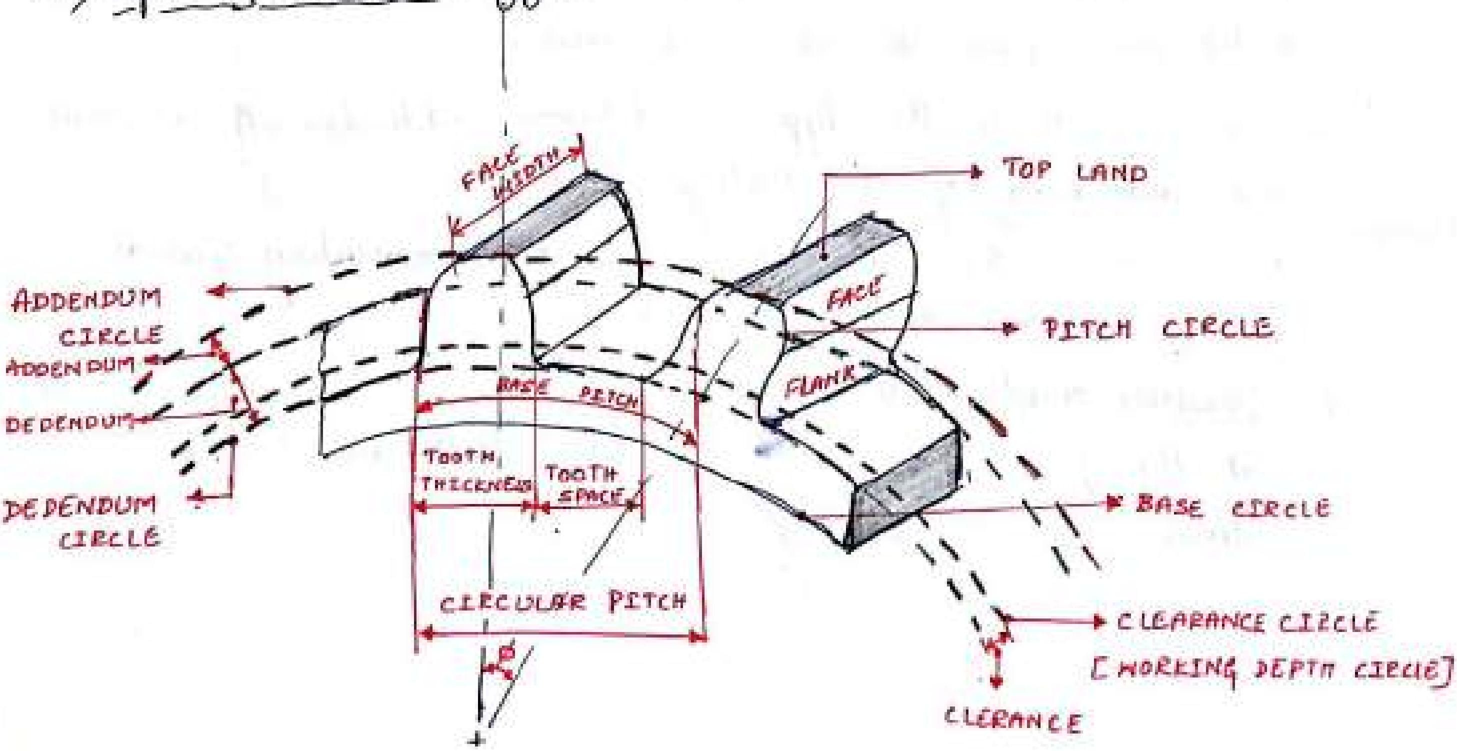
In this gearing, the gears mesh externally & they rotate in opposite direction.

* Spur gear :-

Gears whose teeth are parallel to the centre line of the gear are called spur gear. It is used to transmit motion & power between two parallel shafts.

* Application :- Small watches, gear boxes, machine tools etc.

⇒ Spur gear terminology :-



P.S.V.

i) Circular pitch :- It is the distance between the circumference of the pitch circle from a point on one tooth to a corresponding point on the adjacent tooth.

* It is denoted by "P_c".

So $\langle P_c = \pi m \rangle$ where m = Module.

ii) Module :- It is the length of pitch circle diameter per tooth. It is denoted by "m"

So $\left\langle m = \frac{d}{Z} \right\rangle$ where d = diameter of pitch c/l tooth
Z = number of teeth.

iii) Diametral pitch :- It is number of teeth per unit length on the pitch circle diameter.

It is denoted by "P_d"

So $\left\langle P_d = \frac{Z}{d} \right\rangle$ where Z = no. of teeth
d = pitch cle diameter.

iv) Black lash :- It is the difference between thickness of tooth & width of the tooth space in which it meshes.

v) Clearance :- It is the difference between addendum of one gear and dedendum of the Mating gear.

vi) pitch point :- The point of contact of two Mating gear at pitch circle is known as pitch point.

vii) Pressure angle :- It is the angle between the common normal at the point of contact of two teeth and common tangent to the pitch point.

It is denoted by "ϕ".

viii) Path of contact :- It is the path traced by contact point of a pair of tooth profiles from beginning of engagement to the end of engagement.

ix) Arc of contact :- It is the locus of a point on the pitch circle from the beginning of engagement to the end of engagement of a pair of teeth into mesh.

x) Contact ratio :- It is the ratio of path of contact to the base pitch "P_c"

$$\text{So} \left\langle \text{Contact ratio} = \frac{\text{Path of contact}}{\text{Base pitch}} = \frac{\text{path of contact}}{P_b} \right\rangle$$

xi) pitch angle :- It is the angle subtended by the arc on pitch circle equal in length to the circular pitch.

xii) Base pitch :- It is the distance measured along the base circle from a point on one tooth to the corresponding point on the adjacent tooth.

It is denoted by "P_b"

$$\therefore \left\langle P_b = \pi m \cos\phi = P_c \cos\phi \right\rangle \quad \text{where } \phi \rightarrow \text{pressure angle}$$

xiii) Addendum :- It is the radial height of tooth above pitch circle.

xiv) Dedendum :- It is the radial depth of tooth below pitch circle.

xv) Addendum circle :- The circle which passes through the tip of all tooth is known as addendum circle.

xvi) Dedendum circle :- The circle which passes through the root of all tooth is known as Dedendum circle.

xvii) pitch circle :- An imaginary circle passing through the pitch point having its centre at the axis of the gear.

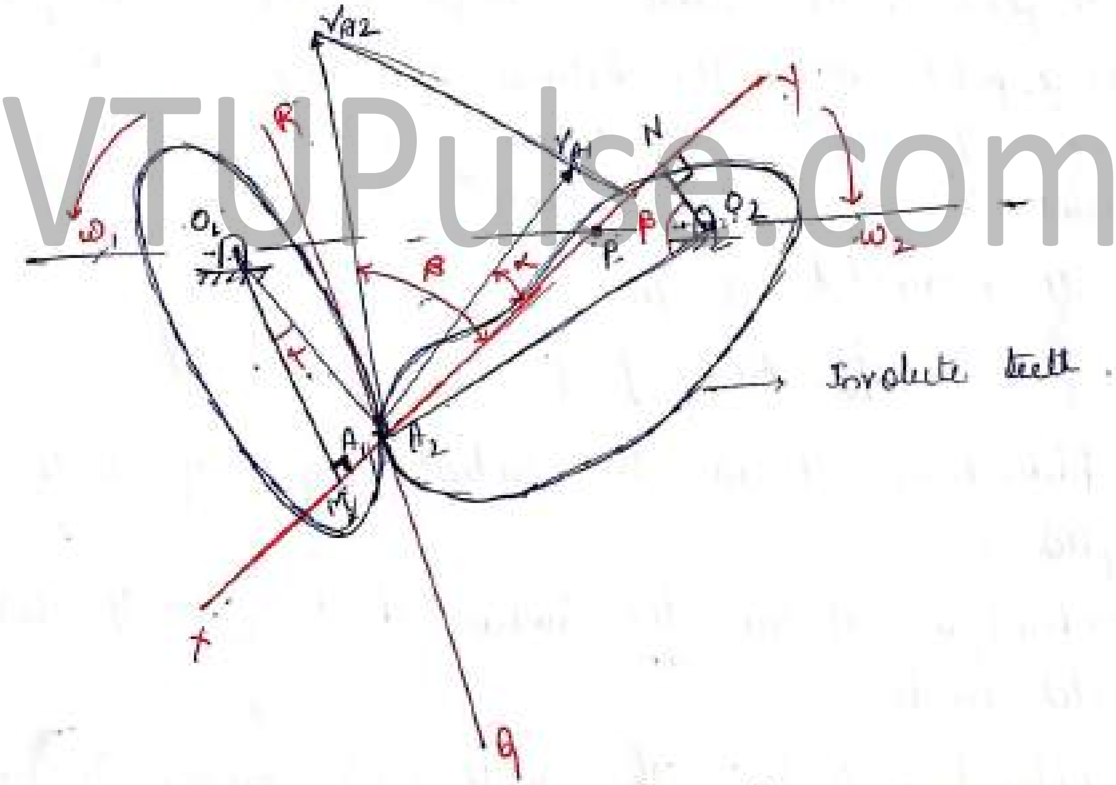
xviii) Face :- The surface above the pitch surface is called the face.

xix) Flank :- The surface below the pitch surface is called the flank.

* **Law of Gearing (or) Condition for correct Gearing :-**

* → Profile :- The curve forming Face & Flank is called as profile.

* Basically 2 types of profiles are used, Mainly involute & cycloid. If the curve is of involute in nature then teeth is called as involute teeth. ||ly If it is cycloidal in shape then the teeth is called cycloidal teeth.



When the tooth profiles are so shaped, and if they produce a constant angular velocity ratio during meshing, then they said to have conjugate action. Involute tooth profile is one of them which gives conjugate action.

⇒ let 2 curved bodies 1 & 2 rotating about centres "O_1" & "O_2" be in contact at "A_1" & "A_2" are coincident points as shown in figure. ④

⇒ "RQ" & "xy" are common tangent at the point of contact respectively.

→ "ω_1" & "ω_2" are the angular velocities of A_1 & A_2 respectively.

→ "V_{A_1}" & "V_{A_2}" are linear velocities at point of contact in the direction perpendicular to "O_1A_1" & "O_2A_2"

⇒ common normal intersect the line joining centres of rotation 1 & 2 at "P".

⇒ "O_1M" & "O_2N" are ⊥er to common normal from O_1 & O_2.

*⇒ If the 2 bodies remain in contact, then component of linear velocities of A_1 & A_2 along common normal (tangent) must be equal.

i.e $V_{A_1} \cdot \cos\alpha = V_{A_2} \cdot \cos\beta$. where V_{A_1} & V_{A_2} → linear velocity

$\therefore \omega_1 \times O_1A_1 \times \cos\alpha = \omega_2 \times O_2A_2 \times \cos\beta$ $\omega_1 O_1A_1$ & $\omega_2 O_2A_2$ → component of L.V

$$\omega_1 \times O_1A_1 \times \frac{O_1M}{O_1A_1} = \omega_2 \times O_2A_2 \times \frac{O_2N}{O_2A_2}$$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \text{Velocity ratio}$$

Also Δ^{le} O_1MP & O_2NP are similar

$$\therefore \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$$\text{Hence velocity ratio} = \frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P}$$

*⇒ Thus for const angular velocity ratio of gearing, the point of contact "P" divides the line joining the centres of rotation in the inverse ratio of angular velocities.

P.S.V

* Hence law of gearing state that "for constant angular velocity ratio of two gear, the common [tangent] normal at point of contact of 2 meshing teeth must pass through fixed point on line joining the centre of rotation."

* Velocity of sliding b/n a pair of involute teeth :-

The velocity of sliding is the velocity of one tooth relative to its mating tooth along common tangent at point of contact.

* component of linear velocity A_1 along common tangent $= V_{A1} \sin\alpha$
* — " — A_2 — " — $= V_{A2} \sin\beta$

linear velocity A_2 relative to linear velocity of A_1 along common tangent $= V_{A2} \sin\beta - V_{A1} \sin\alpha$

$$= \omega_2 \times O_2A_2 \times \frac{A_2N}{O_2A_2} - \omega_1 \times O_1A_1 \times \frac{A_1M}{O_1A_1}$$

$$= \omega_2 \times A_2N - \omega_1 A_1M$$

$$= \omega_2 (A_2P + PN) - \omega_1 \times (PM - A_1P)$$

$$= \omega_2 A_2P + \omega_2 PN - \omega_1 PM + \omega_1 A_2P \quad [\because A_1P = A_2P]$$

$$= (\omega_1 + \omega_2) A_2P + \cancel{\omega_1 PM} - \omega_1 PM \quad \left[\because \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} = \frac{PN}{PM}\right]$$

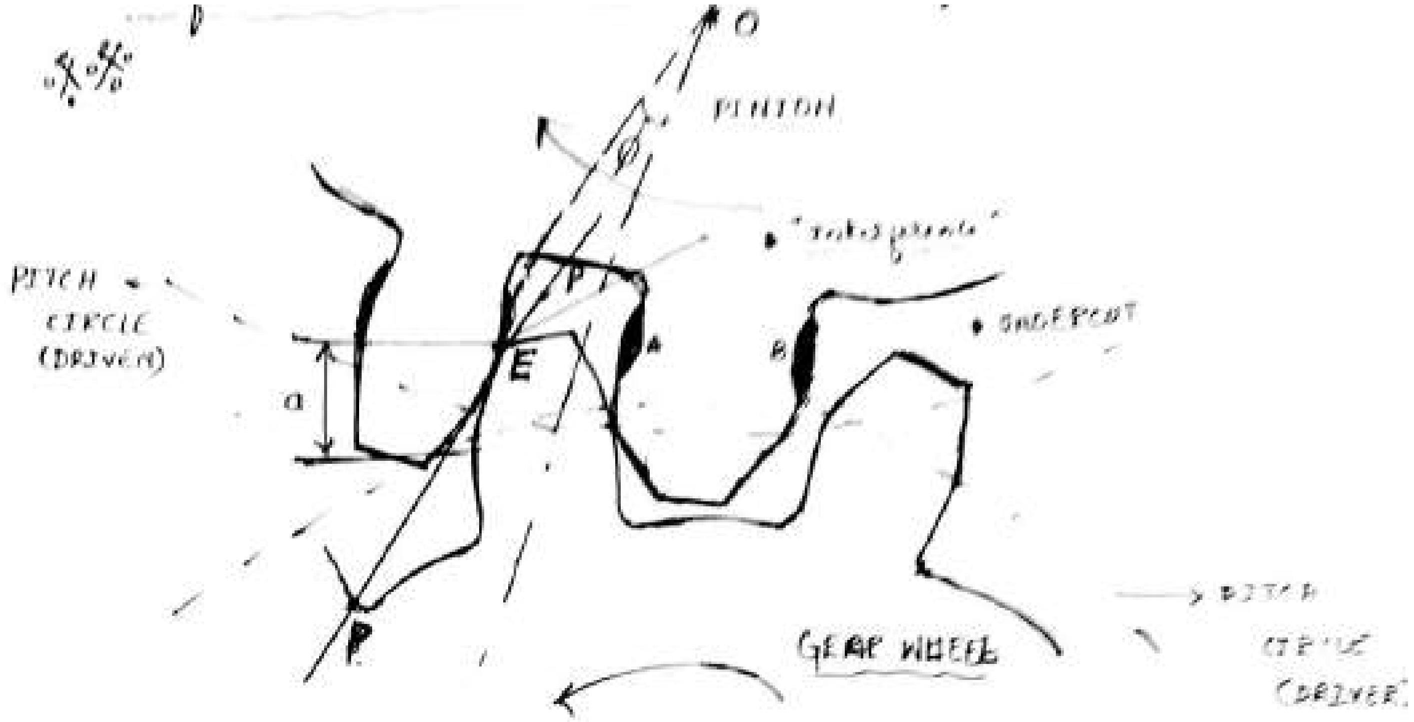
$$[\because PN = PM,\ \omega_1 = \omega_2]$$

$$= (\omega_1 + \omega_2) A_2P$$

$$= (\omega_1 + \omega_2) \cdot A_1P \quad [A_1P = A_2P]$$

⇒ Thus velocity of sliding is equal to the product of the sum of the angular velocities and distance from point of contact.

⇒ Incase of two mating gears, the point of intersection must be the pitch point for correct gearing.



* Defn of Interference:- It is the point of contact of 2 gear teeth when they have to overlap (or) cut into the mating teeth.
(or) Mating of two non-conjugate teeth is known as Interference.

→ In fig shown the pinion is driven by gear wheel driver, & the teeth's of both the gear get meet at point "E", the point is Interference point. Draw the tangent line from point "E" i.e for line of action. & the Max length of line of approach is "EP"

→ The Max addendum of gear which should be used on rack is "a"

→ when gear drives pinion the teeth at point of contact "E" touche (or) comes in contact with other teeth, so during movement the tip of gear teeth takes material from pinion teeth. So wear occur.

→ In order to satisfy the fundamental of law of gearing it is necessary to undercut pinion teeth as shown in fig at point "A" & "B"

* Hence law of gearing states that "for constant angular velocity ratio of two gear, the common [tangent] normal at point of contact ...taking teeth must pass through fixed point on

* The gear to mesh without interference it should satisfy following condition

i) Height of teeth may be reduced.
ii) under cutting is done as shown in fig.
iii) Centre distance may be increased.
iv) By tooth correction. [i.e it should be P/2]

* profile:-

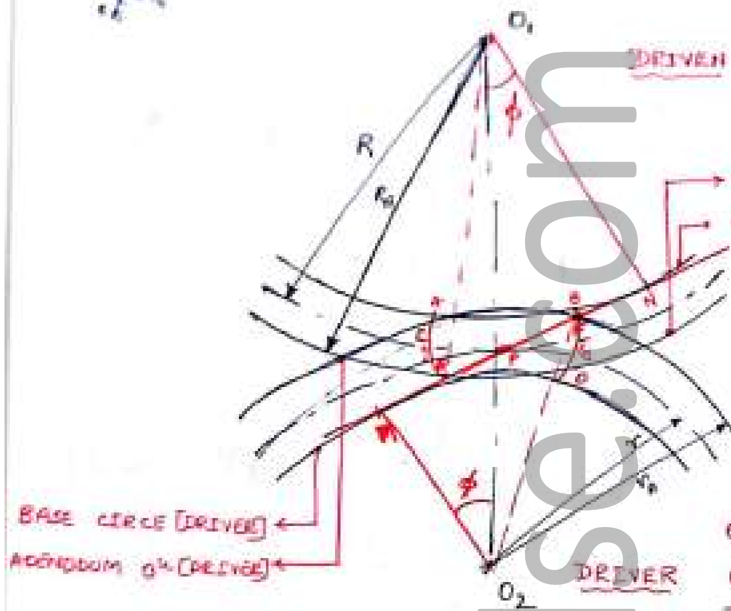
* The curve forming face & flank is called profile.
There are 2 types of profiles mainly:-
1) Involute profile:- The profile formed 'S' shape when 2 teeth comes in contact & moves is called involute teeth.
ii) Cycloidal profile:- The profile formed 'S' shape when 2 teeth comes in contact & moves is called cycloidal teeth.

* Advantages of involute profile:-
1. The pressure angle 'φ' is constant through engagement.
2. A small variation in centre distance doesnot affect velocity ratio.
3. ~~The~~ In involute profile the curve is single curve, so it is easy to manufacture.

* Advantages of cycloidal profile:-
1. In cycloidal profile there is no interference.
2. cycloidal profile have spreading flanks where as involute profile have radial flanks. So cycloidal teeth is stronger.
3. cycloidal profile always have one concave surface will be in contact with convex surface so wear & tear is minimum.

* Length of arc of contact :-

Fig:-



$\phi \rightarrow$ Pressure angle

$CD \rightarrow$ Arc

$GH \rightarrow$ Arc

$O_1P = R \longrightarrow$ radius of pitch c

$O_1E = R_A \longrightarrow$ radius of addendu

$O_2P = r \longrightarrow$ radius of pitch cir

$O_2F = r_A \longrightarrow$ radius of addendu

llly $O_1N \rightarrow$ Base cir radius $= O_1P\cos\phi = R\cos$

& $O_2M \rightarrow$ Base cir radius $= O_2P\cos\phi = r\cos$

Consider driver & driven is meshed as shown in fig. where "MN" is a common tangent to base circle. The addendum circle cut the common tangent MN at points E & F. In other words contact of teeth begins at "E" & ends at "F".

So Length of path of contact, $EF = Ep + PF$. —①

where Ep = path of approach, PF = path of recess

* path of approach :-

i.e $Ep = EN - PN$ —Ⓐ

So from Δ^{le} O_1EN, $EN = \sqrt{O_1E^2 - O_1N^2} = \sqrt{R_A^2 - R^2\cos^2\phi}$ —②

from Δ^{le} O_1PN, $PN = O_1P\sin\phi = R\sin\phi$ —③

P.T.O

So path of approach from ③ & ③ to (*)

we have $\langle E_p = (R_A^2 - R^2\cos^2\phi)^{1/2} - R\sin\phi. \rangle$ —— ④

* path of recess :-

i.e $PF = MF - Mp.$ —— (*)₁

from Δle O_2MF, $MF = \sqrt{O_2F^2 - O_2M^2} = \sqrt{r_A^2 - r^2\cos^2\phi}$ —— ⑤

& from Δle O_2Mp, $Mp = O_2p\sin\phi = r\sin\phi$ —— ⑥

So path of recess from ⑤ & ⑥ to (*)₁

$\langle PF = (r_A^2 - r^2\cos^2\phi)^{1/2} - r\sin\phi. \rangle$ —— ⑦.

Substitute Eqn ④ & Eqn ⑦ in Eqn ①

$EF = [(R_A^2 - R^2\cos^2\phi)^{1/2} - R\sin\phi] + [(r_A^2 - r^2\cos^2\phi)^{1/2} - r\sin\phi]$

$\langle EF = (R_A^2 - R^2\cos^2\phi)^{1/2} + (r_A^2 - r^2\cos^2\phi)^{1/2} - (R+r)\sin\phi \rangle$ —— ⑧

Here arc of contact = Arc CD = Arc GH

∴ Arc CD = $\frac{\text{Arc AB}}{\cos\phi}$

But arc AB = path of contact.

So $\langle \text{Arc of contact} = \frac{\text{length of path of contact}}{\cos\phi} \rangle$

If driver & driven have equal no of teeth, then diameter of the driver & driven gears are same.

∴ $\langle$ length of path of contact = $2[(R_A^2 - R^2\cos^2\phi)^{1/2} - R\sin\phi]$ $\rangle$

Since $R_A = r_A$ & $R = r$

* Contact ratio

The contact ratio is defined as avg no of pair of teeth in contact.

→ This can be found by noting how many time base pitch fits into length of path of contact.

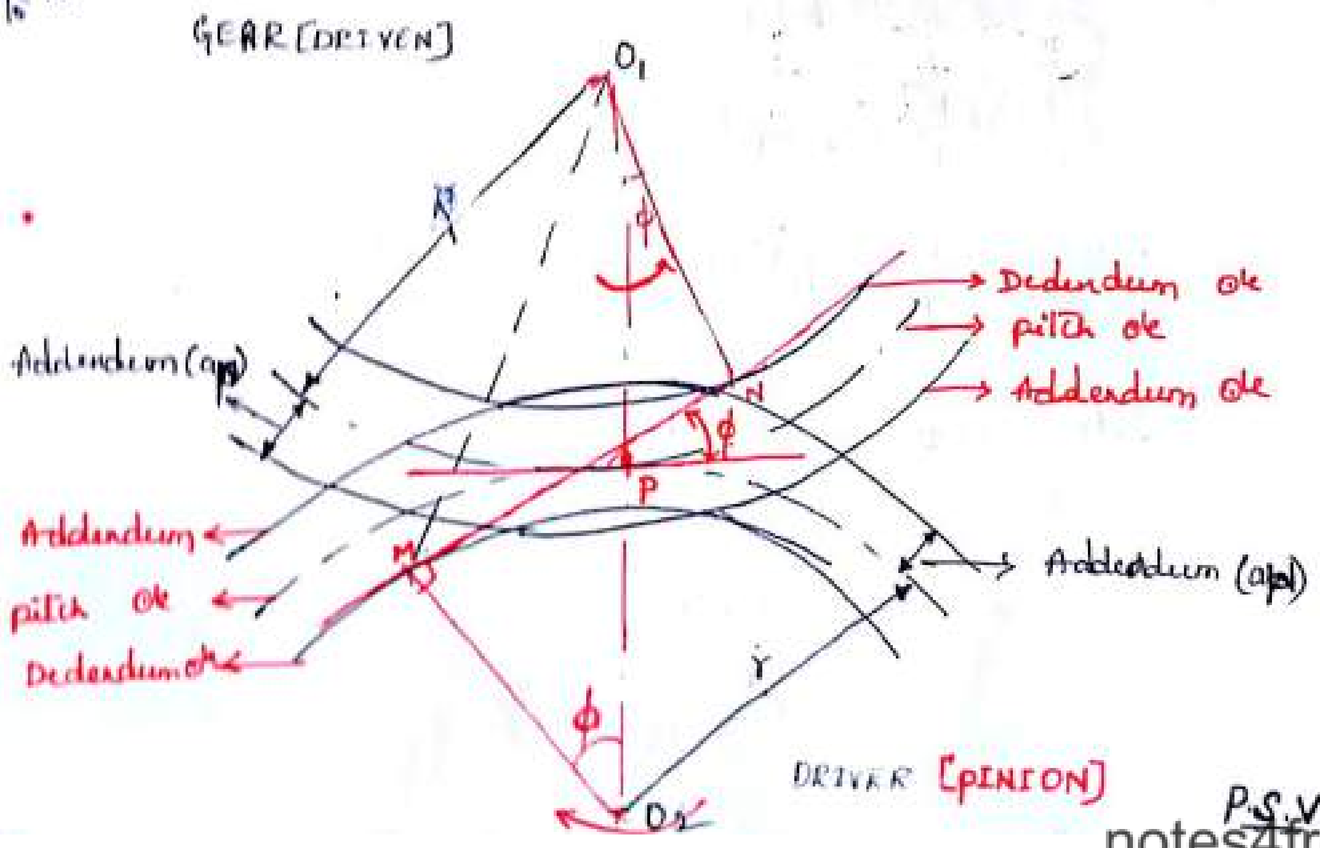
Thus, contact ratio = $\frac{\text{length of path of contact}}{\text{Base pitch}}$

$= \frac{\text{length of path of contact}}{\pi m \cos\phi}$

$= \frac{\text{length of arc of contact}}{\pi m}$

So $\left\langle \text{contact ratio} = \frac{\text{length of arc of contact}}{\text{circular pitch}(p_c)} \right\rangle$

* Minimum no of teeth on a gear to avoid interference & Minimum no of teeth on pinion to avoid interference.



P.S.V.

Let ϕ = pressure angle

R = pitch circle radius of gear $= \frac{mT}{2}$

r = pitch circle radius of driver $= \frac{mt}{2}$

where T = no. of teeth on driven

t = No. of teeth on driver

m = Module

a_w = Addendum coeff of driven

a_p = — of driver

$a_p m$ = Addendum of driver

$a_w m$ = Addendum of driven

G = Gear ratio $= \frac{T}{t}$

Join O_1 to M from the triangle

applying cosine rule.

$$O_1M^2 = O_1P^2 + PM^2 - 2 \cdot O_1P \cdot PM \cdot \cos O_1PM$$

$$= R^2 + r^2\sin^2\phi - 2Rr\sin\phi \cdot \cos(90+\phi) \quad [\because \text{from } O_1PM,\ PM = r\sin\phi]$$

$$= R^2 + r^2\sin^2\phi + 2Rr\sin^2\phi \quad [\cos(90+\phi) = -\sin\phi]$$

$$= R^2\left[1 + \frac{r^2}{R^2}\sin^2\phi + \frac{2r}{R}\sin^2\phi\right]$$

$$= R^2\left[1 + \frac{r}{R}\sin^2\phi\left(\frac{r}{R} + 2\right)\right]$$

$$O_1M = R\left[1 + \frac{r}{R}\sin^2\phi\left(\frac{r}{R} + 2\right)\right]^{1/2} \qquad \left\langle G = \text{Gear ratio} = \frac{T}{t}\right\rangle$$

$$= \frac{mT}{2}\left[1 + \frac{t}{T}\sin^2\phi\left(\frac{t}{T} + 2\right)\right]^{1/2}$$

$$= \frac{mT}{2}\left[1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G} + 2\right)\right]^{1/2} \quad \left[\because G = \frac{T}{t}\right] \qquad \text{--- (1)}$$

Also O_1M = R + addendum of gear.

$$= R + a_w m$$

$$= \frac{mT}{2} + a_w m \qquad \text{--- (2)}$$

Equating ① & ② ⑧

$$\frac{mT}{2} + a_w \cdot m = \frac{mT}{2}\left[1 + \frac{1}{q}\sin^2\phi\left(\frac{1}{q}+2\right)\right]^{1/2}$$

i.e. $$a_w = \frac{T}{2}\left[1 + \frac{1}{q}\sin^2\phi\left(\frac{1}{q}+2\right)\right]^{1/2} - \frac{T}{2}$$

$$a_w = \frac{T}{2}\left[\left\{1 + \frac{1}{q}\sin^2\phi\left(\frac{1}{q}+2\right)\right\}^{1/2} - 1\right]$$

$$\left\langle T = \frac{2a_w}{\left\{1+\frac{1}{q}\sin^2\phi\left(\frac{1}{q}+2\right)\right\}^{1/2} - 1} \right\rangle$$

= No of teeth on gear to avoid interference.

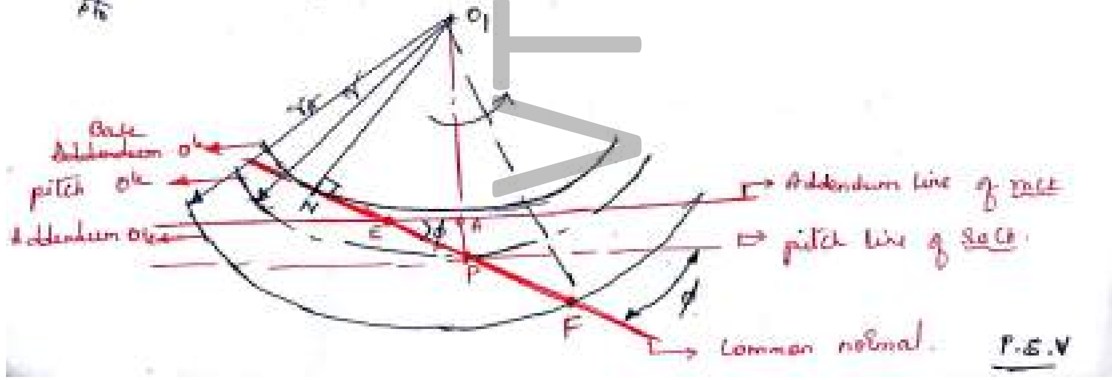
∴ No of teeth on pinion to avoid interference $= t = \frac{T}{q}$

If no of teeth on pinion is same as no of teeth on gear i.e $T = t$, then $q = 1$

$$\left\langle \therefore T = t = \frac{2a_w}{\left\{1+3\sin^2\phi\right\}^{1/2} - 1} \right\rangle$$

* Rack and pinion :-

* Length of path of contact for Rack and pinion :-



P.S.V

where, r = pitch circle radius of pinion $= O_1P$.

r_A = Addendum circle radius of pinion $= O_1F$

a = Addendum of rack

ϕ = Pressure angle.

Assume pinion is driver as shown in fig. The addendum of rack cuts the common normal at point E and the addendum circle of pinion cuts the common normal at "F". i.e contact of teeth begins at "E" and End at "F"

Here length of path of contact = EF = path of approach "EP" + path of recess "PF" —— ①

From Δ^{le} AEP

$$\sin\phi = \frac{AP}{EP} = \frac{a}{EP}$$

$\therefore$ path of approach $= EP = \dfrac{a}{\sin\phi}$ —— ②

$\therefore$ path of recess $= PF = NF - NP$ —— ③

To find NF & NP.

From Δ^{le} O_1NP, $NP = O_1P\sin\phi = r\sin\phi$

$O_1N = O_1P\cos\phi = r\cos\phi$

Join O_1 to F & from Δ^{le} O_1NF

$$NF = \sqrt{O_1F^2 - O_1N^2} = \sqrt{r_A^2 - r^2\cos^2\phi}$$

Substituting the values of NP & NF in eq. ③

$\therefore$ path of recess $= PF = (r_A^2 - r^2\cos^2\phi)^{1/2} - r\sin\phi$ —— ④

Subst ② & ④ in eq ①

we get $\left\langle \text{path of contact} = \dfrac{a}{\sin\phi} + (r_A^2 - r^2\cos^2\phi)^{1/2} - r\sin\phi \right\rangle$

let ϕ = pressure angle

R = pitch circle radius of gear (driven) $= \frac{mT}{2}$

r = pitch circle radius of driver $= \frac{mt}{2}$

where T = no of teeth on driven

t = No of teeth on driver

m = Module.

a_w = Addendum const of driven.

a_p = — " — of driver

$a_w m$ = Addendum of driven

$a_p m$ = Addendum of driver

G = Gear ratio $= \frac{T}{t}$

Join O_1 to M, & from $\triangle^{le}$ O_1MP, applying cosine rule

$$O_1M^2 = O_1P^2 + PM^2 - 2 \times O_1P \times PM \times \cos O_1PM.$$

$$= O_1P^2 + O_2P^2 \sin^2\phi - 2 O_1P \times O_2P \sin\phi . \cos(90+\phi)$$ $[\because \triangle^{le} O_2PM, PM = O_2P\sin\phi]$

$$= O_1P^2 + O_2P^2 \sin^2\phi + 2 O_1P \times O_2P \sin^2\phi$$ $[\because \cos\phi = -\sin\phi]$

$$= R^2 + r^2 \sin^2\phi + 2Rr\sin^2\phi.$$

$$= R^2\left[1 + \frac{r^2}{R^2}\sin^2\phi + \frac{2r}{R}\sin^2\phi\right]$$

$$= R^2\left[1 + \frac{r}{R}\sin^2\phi\left(\frac{r}{R}+2\right)\right]$$

$$O_1M = R\left[1 + \frac{r}{R}\sin^2\phi\left(\frac{r}{R}+2\right)\right]^{1/2}$$ $\langle G = \text{Gear ratio} = \frac{T}{t}\rangle$

$$= \frac{mT}{2}\left[1 + \frac{t}{T}\sin^2\phi\left(\frac{t}{T}+2\right)\right]^{1/2}$$

$$= \frac{mT}{2}\left[1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G}+2\right)\right]^{1/2} \quad \left[\because G = \frac{T}{t}\right]. \quad \text{—— (1)}$$

Also $O_1M = O_1P$ + addendum of gear.

$$= R + a_w m,$$

$$= \frac{mT}{2} + a_w m. \quad \text{—— (2)}$$

where, r = pitch circle radius of pinion $= O_1P$,
r_A = Addendum circle radius of pinion $= O_1F$
a = Addendum of rack
ϕ = pressure angle.

Assume pinion is driven as shown in fig. The addendum of rack cuts the common normal at point E and the addendum circle of pinion cuts the common normal at "F". i.e contact of teeth begins at "E" and end at "F".

Here length of path of contact = EF = path of approach "Ep" + path of recess "PF" —— ①

From Δ^{le} AEP

$$\sin\phi = \frac{AP}{EP} = \frac{a}{EP}$$

$\therefore$ path of approach $= EP = \frac{a}{\sin\phi}$ —— ②

$\therefore$ path of recess $= PF = NF - NP$ —— ③

To find NF & NP.

From Δ^{le} O_1NP, $NP = O_1P\sin\phi = r\sin\phi$

$O_1N = O_1P\cos\phi = r\cos\phi$

Join O_1 to F & from Δ^{le} O_1NF

$$NF = \sqrt{O_1F^2 - O_1N^2} = \sqrt{r_A^2 - r^2\cos^2\phi}$$

Substituting the values of NP & NF in eq. ③

$\therefore$ path of recess $= PF = (r_A^2 - r^2\cos^2\phi)^{1/2} - r\sin\phi$ —— ④

Subst ② & ④ in eq ①

we get $\left\langle \text{path of contact} = \frac{a}{\sin\phi} + (r_A^2 - r^2\cos^2\phi)^{1/2} - r\sin\phi \right\rangle$

Equating ① & ②

$$\frac{mT}{2} + a_w \cdot m = \frac{mT}{2}\left[1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G} + 2\right)\right]^{1/2}$$

i.e $a_w = \frac{T}{2}\left[1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G} + 2\right)\right]^{1/2} - \frac{T}{2}$.

$$a_w = \frac{T}{2}\left[\left\{1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G} + 2\right)\right\}^{1/2} - 1\right]$$

$$T = \frac{2a_w}{\left\{1 + \frac{1}{G}\sin^2\phi\left(\frac{1}{G} + 2\right)\right\}^{1/2} - 1}$$

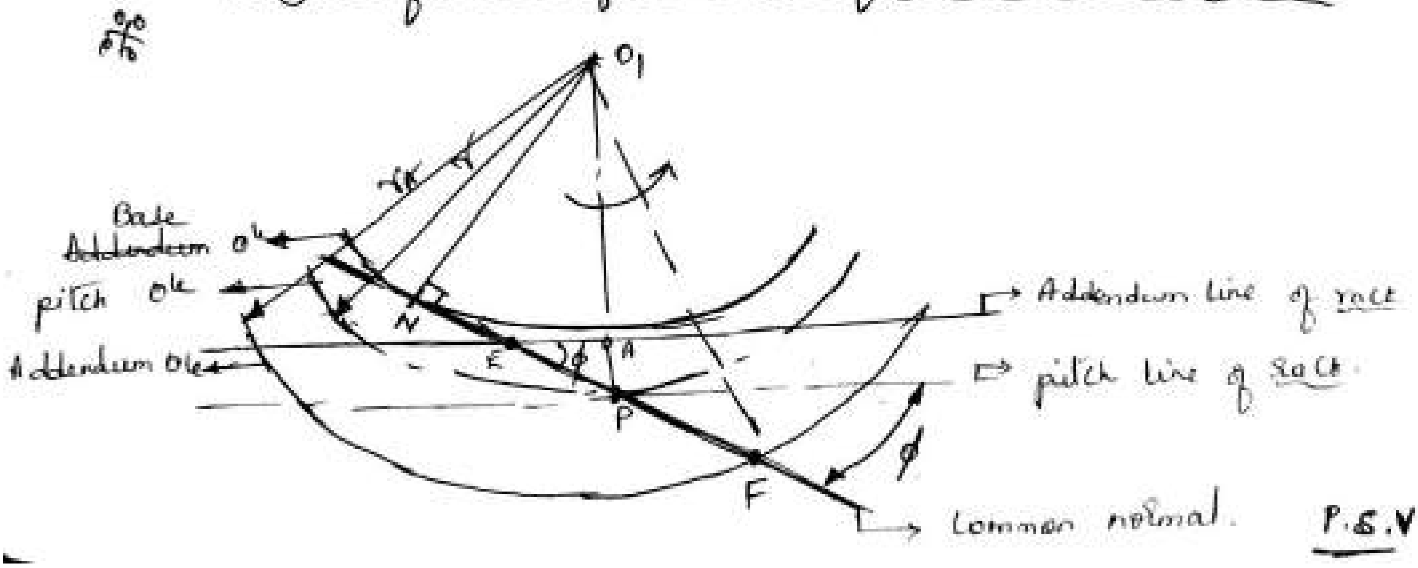
= No of teeth on gear to avoid interference.

∴ No of teeth on pinion to avoid interference $= t = \frac{T}{G}$

If no of teeth on pinion is same as no of teeth on gear i.e $T = t$, then $G = 1$

$$\therefore \quad T = t = \frac{2a_w}{\left\{1 + 3\sin^2\phi\right\}^{1/2} - 1}$$

* Rack and pinion :-
* Length of path of contact for Rack and pinion :-



P.T.O

* Minimum no of teeth for pinion on Rack:-

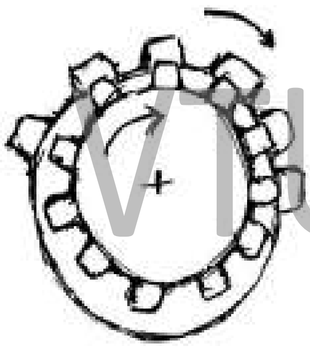
Min no of teeth on pinion.

$$t = \frac{2 \cdot a_r}{\sin^2 \phi}$$

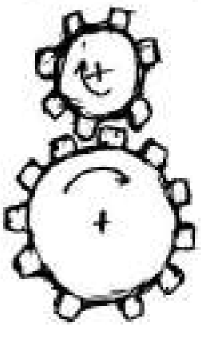
where a_r = addendum constant of rack.
a = addendum of rack = $a_r \cdot m$.
m = Module.

* Internal & External Spur Gear:-

\+ Internal gearing: fig shows the spur gear with ~~External~~ Internal teeth on the ~~outer~~ inner surface of cylinder.
Hence shaft rotate in the ~~opposite~~ same directions.

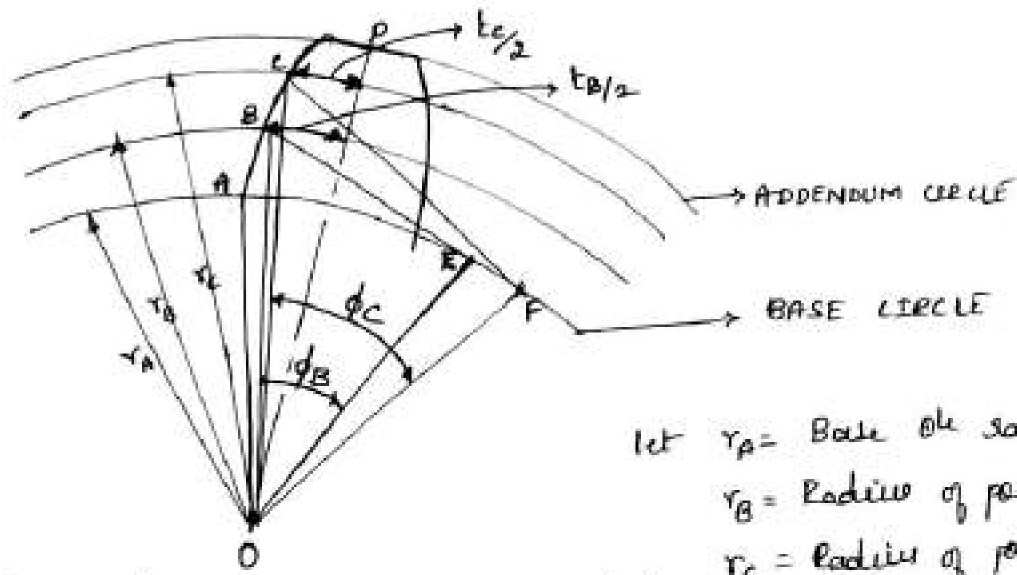


\+ External gearing:- fig shows the spur gear with External teeth on the outer surface of cylinder
Hence shaft rotate in opposite directions.



P.S.V

* Involutometry:-



let r_A = Base circle radius.
r_B = Radius of point B
r_C = Radius of point C.
ϕ_B = pressure angle for point B
ϕ_C = pressure angle for point C
t_B = Arc tooth thickness at B
t_C = Arc tooth thickness at C

* The study of geometry of involute is called involutometry.
* Consider an involute of base circle radius r_A
let "B" & "C" be two points on involute.
* Draw normal to involute from point B & C.
The normal BE & CF are tangent to base circle.

⇒ From Δle OBE

$$\cos\phi_B = \frac{OE}{OB} = \frac{r_A}{r_B} \quad \text{—} ①$$

$$\langle \therefore r_A = r_B\cos\phi_B \rangle$$

⇒ From Δle OCF

$$\cos\phi_c = \frac{OF}{OC} = \frac{r_A}{r_C} \quad \text{—} ②$$

$$\langle \therefore r_A = r_C\cos\phi_c \rangle$$

Equating ① & ②, we get

$$r_B\cos\phi_B = r_C\cos\phi_c \quad \text{—} ③$$

* From properties of involute
Arc AE = length BE
Arc AF = length CF

* From fig

$$A\hat{O}E = \frac{\text{Arc } AE}{OE} = \frac{BE}{OE} = \tan\phi_B$$

$$A\hat{O}B = A\hat{O}E - \phi_B = \tan\phi_B - \phi_B$$

$$\text{i.e} \langle inv\,\phi_B = \tan\phi_B - \phi_B \rangle \quad \text{—} ④$$

⇒ $\tan\phi_B - \phi_B$ is called as an involute function.

* From fig

$$A\hat{O}F = \frac{\text{Arc } AF}{OF} = \frac{CF}{OF} = \tan\phi_c$$

$$A\hat{O}C = A\hat{O}F - \phi_c = \tan\phi_c - \phi_c$$

$$\text{i.e} \langle inv\,\phi_c = \tan\phi_c - \phi_c \rangle \quad \text{—} ⑤$$

from fig for point B

$$A\hat{O}D = A\hat{O}B + \frac{t_B}{2r_B}$$

$$\left\langle A\hat{O}D = \tan\phi_B - \phi_B + \frac{t_B}{2r_B} \right\rangle \quad \text{——— (6)}$$

lll'y for the point C

$$A\hat{O}D = A\hat{O}C + \frac{t_c}{2r_c}$$

$$\left\langle A\hat{O}D = \tan\phi_C - \phi_C + \frac{t_C}{2r_C} \right\rangle \text{——— (7)}$$

Equating (6) & (7)

$$\tan\phi_c - \phi_c + \frac{t_c}{2r_c} = \tan\phi_B - \phi_B + \frac{t_B}{2r_B}$$

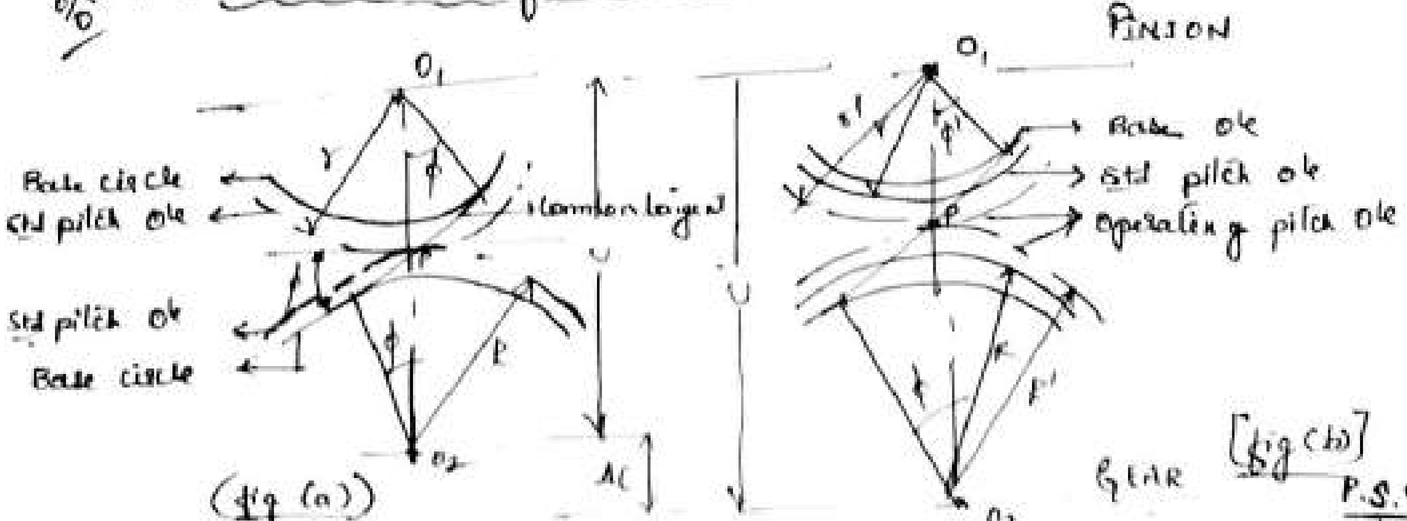
$$\text{inv}\,\phi_c + \frac{t_c}{2r_c} = \text{inv}\,\phi_B + \frac{t_B}{2r_B}$$

$$\frac{t_c}{2r_c} = \text{inv}\,\phi_B - \text{inv}\,\phi_c + \frac{t_B}{2r_B}$$

$$\left\langle \therefore t_c = \left(\text{inv}\,\phi_B - \text{inv.}\,\phi_c + \frac{t_B}{2r_B}\right) 2 \cdot r_c \right\rangle$$

$t_c \rightarrow$ tooth thickness at C

% * Determination of Backlash:-



(fig (a)) [fig (b)]

P.S.V

fig (a) shows 2 standard gears meshing at centre distance c. III^ly fig (b) shows condition after the 2 gears have been pulled apart at a centre distance 'Δc' to give new centre distance c'. Let p' be the new pitch point.

let r = Std pitch circle radius of pinion.
R = ——— " ——— gear.
} c = Std centre distance $= r+R$

r' = operating pitch cle radius of pinion
R' = ——— " ——— gear.
} c' = operating centre distance $= r'+R'$

ϕ = pressure angle standard,
ϕ' = operating pressure angle.

h = tooth thickness of pinion on Std pitch cle $= \frac{p}{2}$

h' = tooth thickness of pinion on operating pitch cle

H = tooth thickness of gear on Std pitch circle.

H' = ——— " ——— on operating pitch circle.

p = Std circular pitch $= \frac{2\pi r}{t} = \frac{2\pi R}{T}$

p' = operating pitch circle $= \frac{2\pi r'}{t} = \frac{2\pi R'}{T}$

Δc = change in centre distance.

B = Backlash

t = no of teeth on pinion.

T = no of teeth on gear.

We know, By velocity ratio $\frac{r}{r'} = \frac{R}{R'} = \frac{c}{c'}$

& also $c'\cos\phi' = c\cos\phi$

$\therefore \left\langle c' = c\frac{\cos\phi}{\cos\phi'} \right\rangle$

Here $\Delta c = c'-c = c\frac{\cos\phi}{\cos\phi'} - c = C\left[\frac{\cos\phi}{\cos\phi'} - 1\right]$

On the operating pitch circle.

operating pitch = sum of tooth thickness + Backlash

$p' = h' + H' + B$ —— (*)

By involutometry $h' = 2r'\left[\text{inv}\phi - \text{inv}\phi' + \frac{h}{2r}\right]$ —— (1)

$\text{lly}\ H' = 2R'\left[\text{inv}\phi - \text{inv}\phi' + \frac{h}{2R}\right]$ —— (2).

substituting the values of h' & H' in Eq. (*)

we get $p' = 2r'\left[\text{inv}\phi - \text{inv}\phi' + \frac{h}{2r}\right] + 2R'\left[\frac{h}{2R} + \text{inv}\phi - \text{inv}\phi'\right] + B$

$p' = h\left[\frac{r'}{r} + \frac{R'}{R}\right] + 2\,\text{inv}\,\phi\,(r' + R) - 2\,\text{inv}\,\phi'(r' + R') + B$

$p' = h\left[\frac{c'}{c} + \frac{c'}{c}\right] + 2c'\,\text{inv}\,\phi - 2c'\,\text{inv}\,\phi' + B$

$\therefore B = p' - 2h\cdot\frac{c'}{c} + 2c'\left[\text{inv}\phi' - \text{inv}\,\phi\right]$

$= \frac{2\pi r'}{t} - 2\cdot\frac{2\pi r}{2t}\cdot\frac{c'}{c} + 2c'\left[\text{inv}\phi' - \text{inv}\,\phi\right] \quad \left(\because h = \frac{p}{2}\right)$

$= \frac{2\pi}{t}\left(r' - r\cdot\frac{c'}{c}\right) + 2c'\left[\text{inv}\phi' - \text{inv}\phi\right]$

$= \frac{2\pi}{t}\left[r' - \cancel{r}\cdot\frac{r'}{\cancel{r}}\right]^{0} + 2c'\left[\text{inv}\phi' - \text{inv}\phi\right] \quad \left(\because \frac{r}{r'} = \frac{c'}{c}\right)$

$= 2c'\left[\text{inv}\phi' - \text{inv}\phi\right]$

$$\left\langle B = 2c'\left[\text{inv}\,\phi' - \text{inv}\,\phi\right]\right\rangle$$

---

* Refer. J.B.K. Das problems for Examination.

* Important :- ~~All~~ Solve all the Question paper problem.

P.S.V.

* Definition :- A cam is a Mechanical Member (or) Machine Element which is used for transmitting a desired Motion that may be reciprocating (or) oscillating to the follower by direct contact.

⇒ The cam & the follower form's a point contact, line contact so this is a higher pair.

* Application :- Operating the inlet & Exhaust Valves of I.C. Engines, Spinning Machines, paper cutting Machine, Machine tools etc.

The simplest cam Mechanism consists of cam, follower & frame.

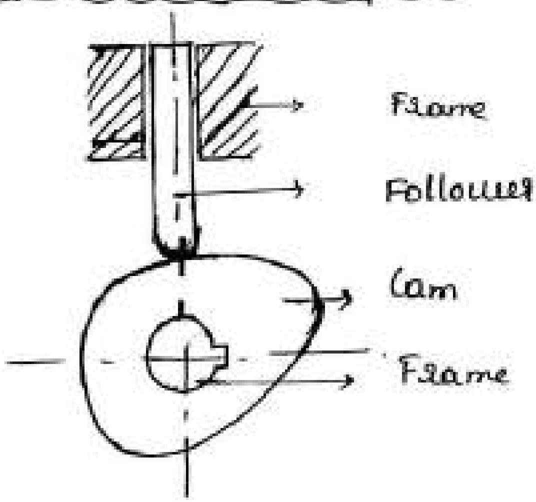
* Cam :- A driver Member known as the Cam.
* Follower :- A driven member called the follower.
* Frame :- A frame which supports the cam & guides the follower.

* Classification of follower :-

Followers can be classified base on

i) According to the Surface of contact.
ii) According to the Motion of the follower.
iii) According to the position of the follower.

* Simple cam Mechanism parts :-

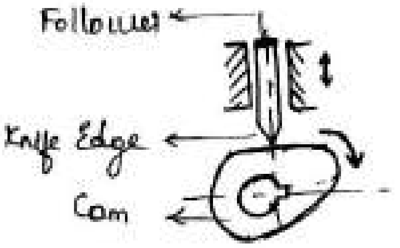


P.S.V

i) According to the surface of contact:

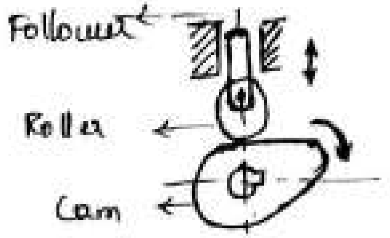
We have 4 types of follower a/c to surface of contact, Mainly,

a) Knife Edged follower



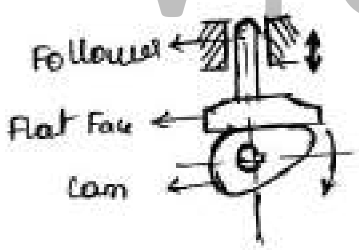
The driven Member i.e follower is in the shape of knife edge at the end & it has point contact with cam.

b) Roller follower



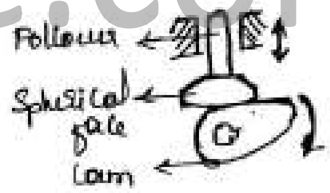
The driven Member i.e follower is having roller at the End & it has point contact with cam

c) Flat Faced follower



The driven Member i.e follower is having flat faced shape at the End & it has line contact & point contact with cam

d) Spherical faced follower



The driven Member i.e follower is having curved faced shape at the End & it has point & line contact with cam.

ii) According to the Motion of the follower:

There are 2 types of follower a/c to motion, Mainly.

a) Translation (or) reciprocating follower.

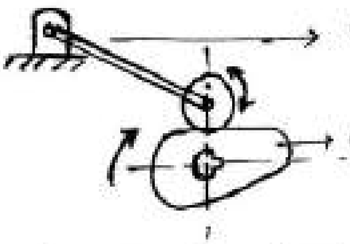
b) Oscillation (or) oscillating follower.

a) Translation (or) reciprocation follower :-

The follower reciprocates as the cam rotates uniformly.

Eg :- Knife edge, Roller, flat face, spherical face follower.

b) Oscillation (or) oscillating follower :-



The follower is pivoted at a suitable point & oscillates as the cam makes the rotary motion

iii) According to the position of the follower :-

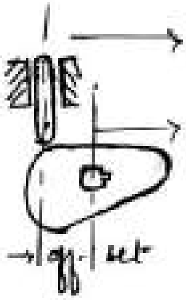
There are 2 types of follower a/c to the position, Mainly :-

a) Radial follower b) off-set follower.

* Radial follower :- The axis of the follower passes through the centre of cam.

Eg : Knife Edge, Roller, flat face, spherical face follower.

* off-set follower :- The axis of the follower movement is displaced from the cam centre axis.



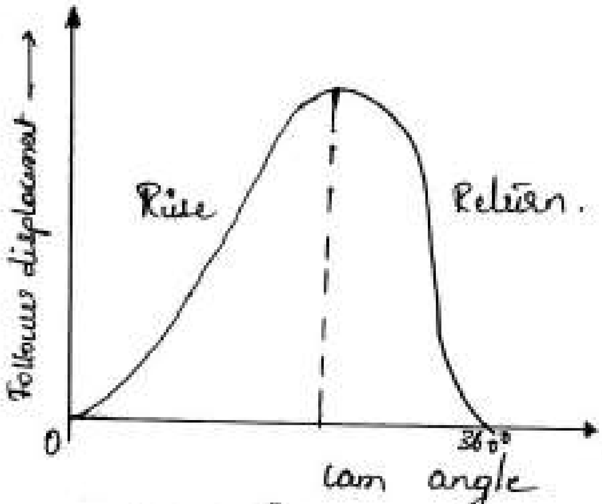
* Classification of Cam's :-

There are 3 types of cam's, Mainly :-

1) Rise - Return - Rise [R-R-R] Cam.
2) Dwell - Rise - Return - Dwell [D-R-R-D] Cam.
3) Dwell - Rise - Dwell - Return - Dwell [D-R-D-R-D] Cam.

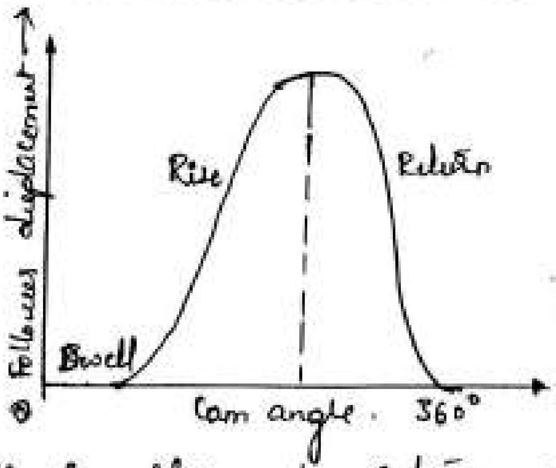
* <u>Dwell</u>:- A Dwell is the zero displacement (or) the absence of motion of follower during motion of the cam. ⑤

1) <u>Rise - Return - Rise [R-R-R] Cam</u>:-



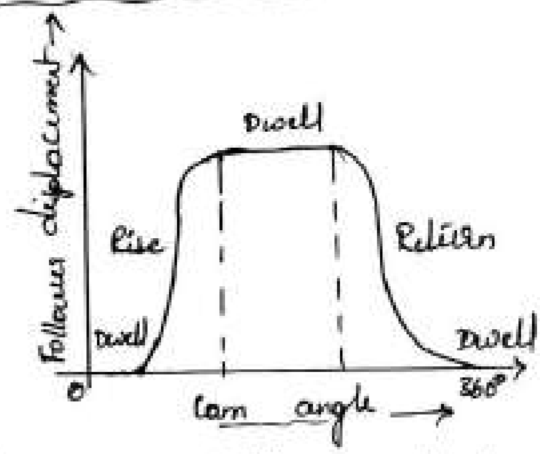
In this there is alternate rise & return of the follower with no periods of dwells. Its use is very limited in the industry.

2) <u>Dwell - Rise - Return - Dwell [D-R-R-D] Cam</u>:-

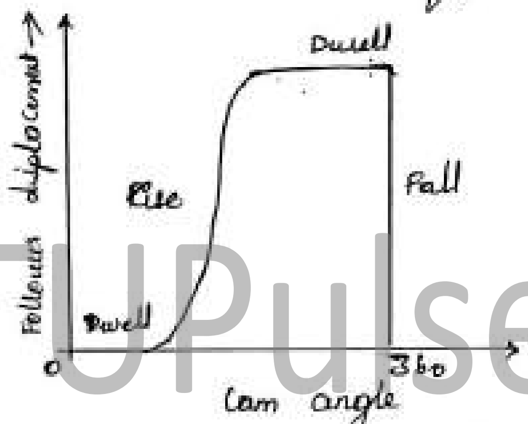


In this cam there is rise and return of the follower after a dwell. This type is used more frequently than R-R-R type of cam.

3) Dwell - Rise - Dwell - Return - Dwell (D-R-D-R-D) Cam :-



It is the most widely used type of cam. The dwelling of the cam is followed by rise & dwell & subsequently by return & dwell as shown in fig.



In case the return of the follower is by fall, the motion may be known as Dwell-Rise-Dwell (D-R-D)

* classification of cam based on shape of the cam:-

There are 3 types of cams based on shape, mainly :-

i) wedge (or) flat cam ii) Radial (or) disc cam

iii) cylindrical (or) drum cam.

i) wedge (or) Flat cam:-

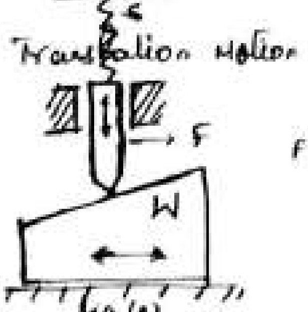


fig (a)

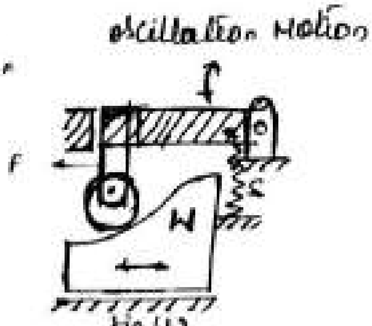
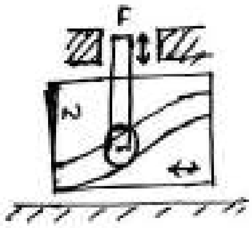


fig (b)

F → Follower
W → wedge (cam)
S → spring

A wedge cam has a wedge 'W' which in general has a translational motion (or) oscillation motion.
When cam rotates the follower translate & oscillates in fig (a) & fig (b) respectively. A spring is usually used to maintain the contact between the cam & the follower.

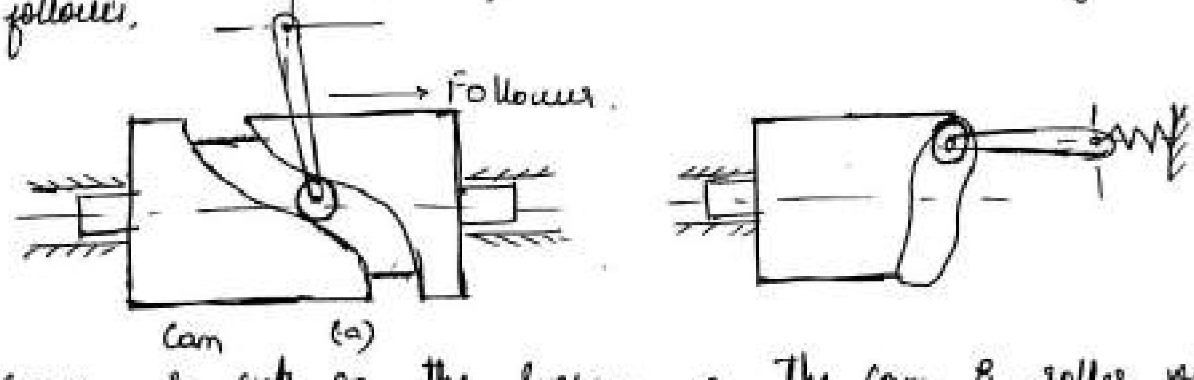


F → Follower
W → Wedge.

Instead of using a wedge, a flat plate with a groove can also be used. Thus a positive drive is achieved without the use of spring.

ii) Radial (or) disc cam:- A cam in which the follower moves radially from the centre of rotation of the cam is known as a radial (or) disc cam.
The follower is held in position by spring (or) gravity.
Eg:- Knife edge, Roller, Flat, spherical, (Translat) etc (or) belongs to this category.

iii) cylindrical (or) drum cam:-
It has a circumferential contour cut in surface, rotates about its axis. The follower motion can be two types as follows,



i) groove is cut on the surface of the cam & roller follower has constrained (positive) oscillating motion.
ii) Another type is an end cam in which the end of the cylinder is the working surface. Follower have translatory motion.
+ cylindrical cam are also known as barrel (or) drum cams.

iii) <u>classification based on constraint</u> :-

constraint may be obtained either by spring loading to keep the follower in contact with cam surface or by positive drive.

- There are 2 types based on constraint mainly:-

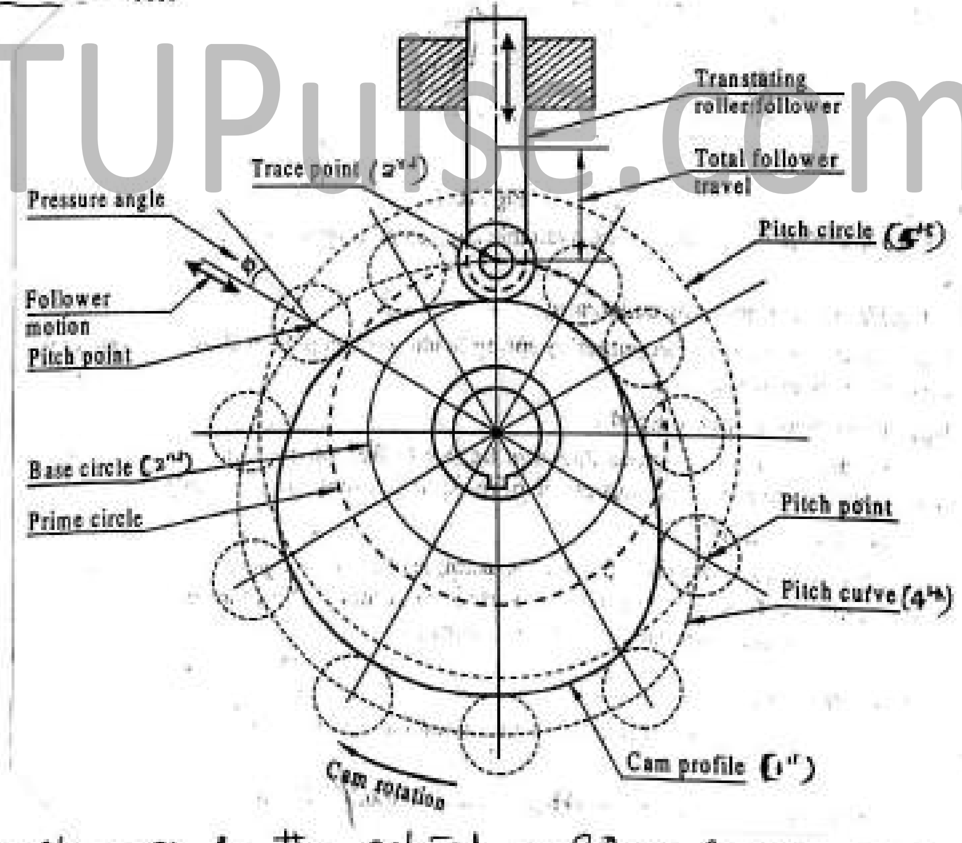
a) <u>Spring loaded (or) pre-loaded</u> :-

In the cam, the follower is to be held by an external force provided by the spring or gravity.

b) <u>positive constraint</u> :-

The cylindrical cam which is not required to keep the follower & the cam surface in contact.

- <u>Cam terminology</u> :-



i) <u>Cam profile</u> :- It is the actual working surface contour of the cam.

ii) <u>Base circle</u> :- It is the smallest circle drawn to the cam profile from the cam centre.

* Trace point :- It is the point on the follower located at the knife edge in knife edge follower & the centre of roller follower (or) centre of spherical face.
* pitch curve :- It is the path of trace point.
* lift (or) stroke :- It is the maximum displacement of the follower from its lowest position to the topmost position.
* pressure angle :- It is the angle b/n the normal to the pitch curve & the instantaneous direction of the follower motion.
* pitch point :- It is the point on the pitch curve having the maximum pressure angle.
* Dwell :- It is the period during which follower is at rest.
* pitch circle :- It is the circle from the cam centre through the pitch points.
* prime circle :- It is the smallest circle drawn to the pitch curve from the centre of rotation of cam.
* Cam angle :- It is the angle of rotation of the cam for a definite displacement of the follower.

Displacement diagram :-

A displacement diagram is a graph displacement of the follower plotted as a function of time.

* Degrees of cam rotation are plotted along the horizontal axis & the length of diagram represent one revolution of the cam.
* Since cam rpm is constant, equal angular division also represent equal time increments.
* Displacement of the follower is plotted along the vertical axis.
* Displacement diagram determines the shape of the cam.
* Since diagram is in reality a displacement v/s time graph. 11ly velocity v/s time & acceleration v/s time graph.

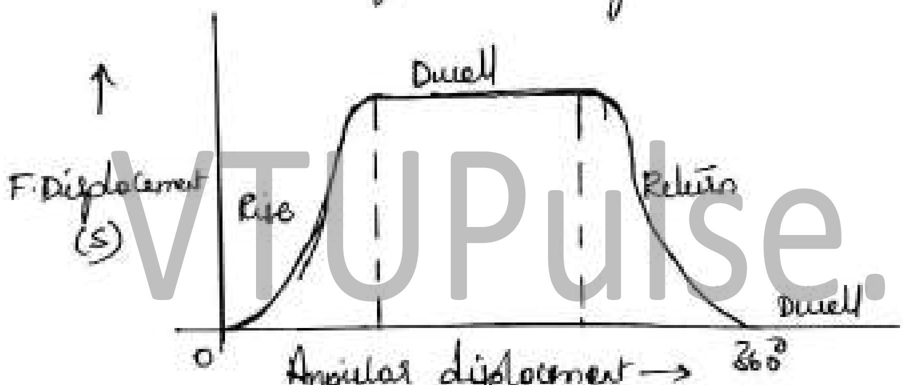
Type of follower motion :-

The follower during its travel may have one of the following type of motion.

i) uniform velocity.
ii) uniform acceleration & retardation (UARM)
iii) simple Harmonic Motion (SHM)
iv) Cycloidal Motion.

* i) Uniform Velocity :-

Displacement, velocity & Acceleration diagram when follower Moves with uniform velocity.



* ii) uniform acceleration & retardation (UARM)

The displacement diagram when a follower moves with UARM.

Constructions steps :-

i) on horizontal line Mark $AB = \theta_a$ = Angle of rise
lly BC = Angle of dwell
$CD = \theta_d$ = Angle of return (decent)
DE = Angle of dwell, for suitable scale

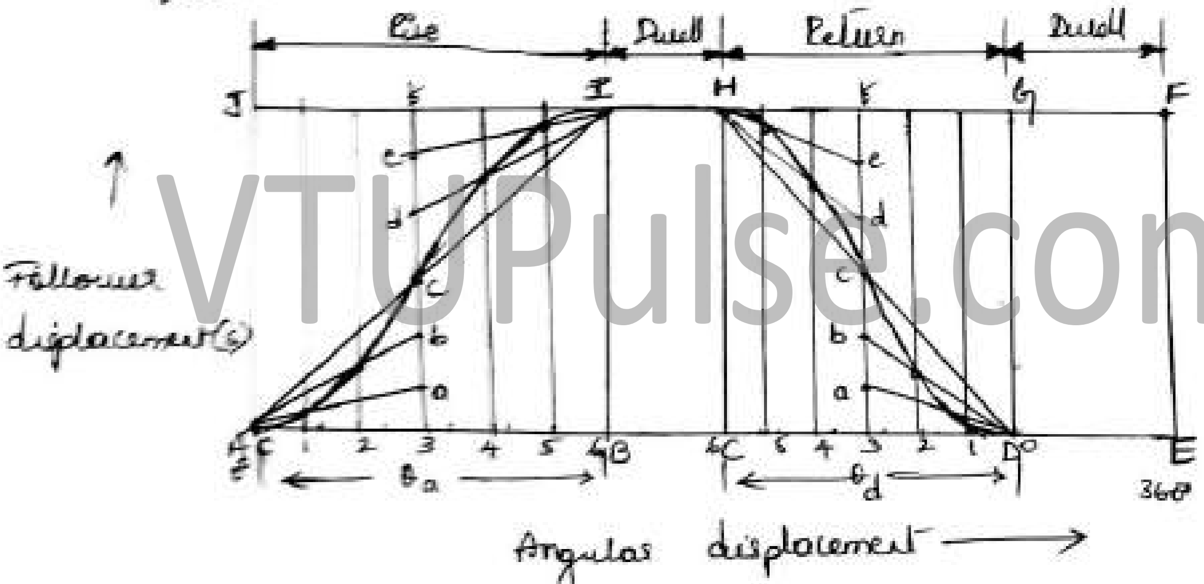
ii) From A draw ⊥^{r} AJ = S = displacement.
Now complete AEFJ rectangle. III^{ly} (draw) B, C, D draw ⊥^{r}s from BI, CH & DG respectively.

iii) Divide AB into even equal parts (Say 6 here) & divide middle line(s) into same no of equal parts.

iv) Join A to a, A to b, A to C
IIIly I to c, I to d, I to e & I to f.

v) Join all these points to get parabolic curve for outstroke [rise] of the follower.

vi) In the same manner, draw displacement diagram for inward [return] stroke.



*Formula to calculate "Max velocity" & "Max Acceleration":-

i) Out stroke (Rise)

Velocity: $V_a = \frac{2\omega S}{\theta_a}$

Acceleration: $a_a = \frac{4\omega^2 S}{\theta_a^2}$

In stroke (Return)

$V_d = \frac{2\omega S}{\theta_d}$

$a_d = \frac{4\omega^2 S}{\theta_d^2}$

where S = stroke (or) follower displacement.

ω = angular velocity = $\frac{2\pi N}{60}$

iv) Cycloidal Motion :-

Displacement, velocity & acceleration diagrams when follower moves with cycloidal motion.

* Construction Steps :-

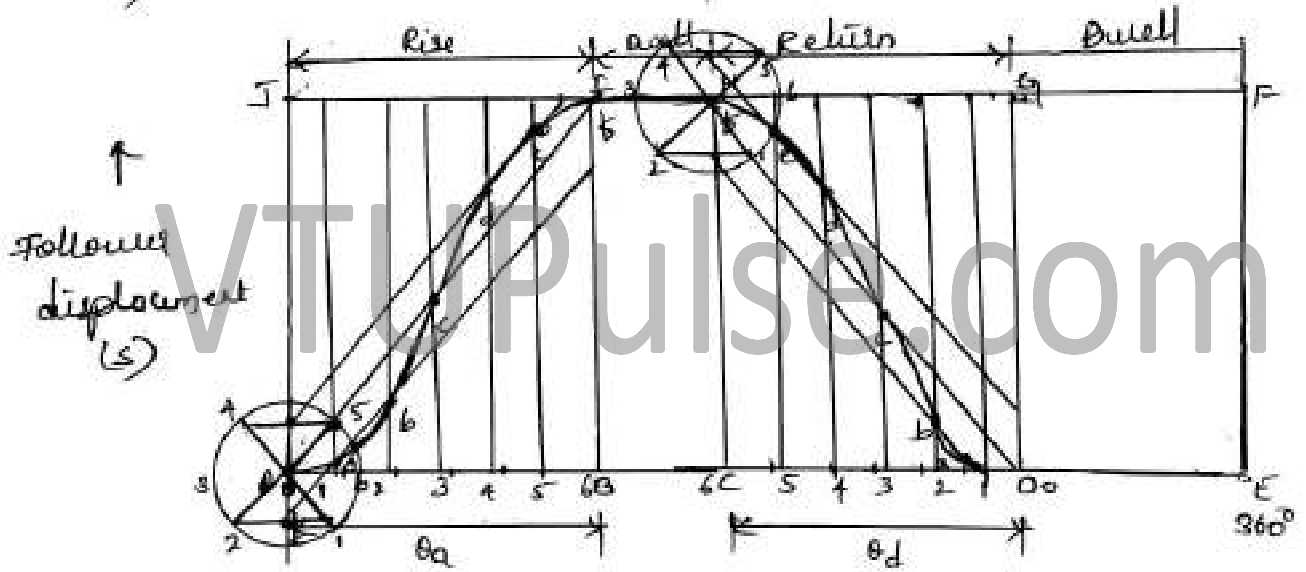
i) Follow the same steps i) ii) iii) as it is of SHM, & UARM.

ii) Cycloidal :- It is the locus of a point on a circle which is rolled on a straight line without slipping.

Draw the circumference with 'r' as radius, 's' as stroke,

so radius $r = \frac{S}{2\pi}$ $[\because S = 2\pi r]$

iii) Divide the circle into same equal part i.e "6"



* Formulas for Calculating Max Velocity & Max Acceleration :-

	Max Velocity	Max Acceleration
Out stroke	$V_a = \frac{2\omega S}{\theta_a}$	$a_a = \frac{2\pi \omega^2 S}{\theta_a^2}$
In stroke	$V_d = \frac{2\omega S}{\theta_d}$	$a_d = \frac{2\pi \omega^2 S}{\theta_d^2}$

iii) Simple Harmonic Motion :- (SHM)

Displacement, velocity and Acceleration diagram when follower moves with SHM

+ Construction Steps :-

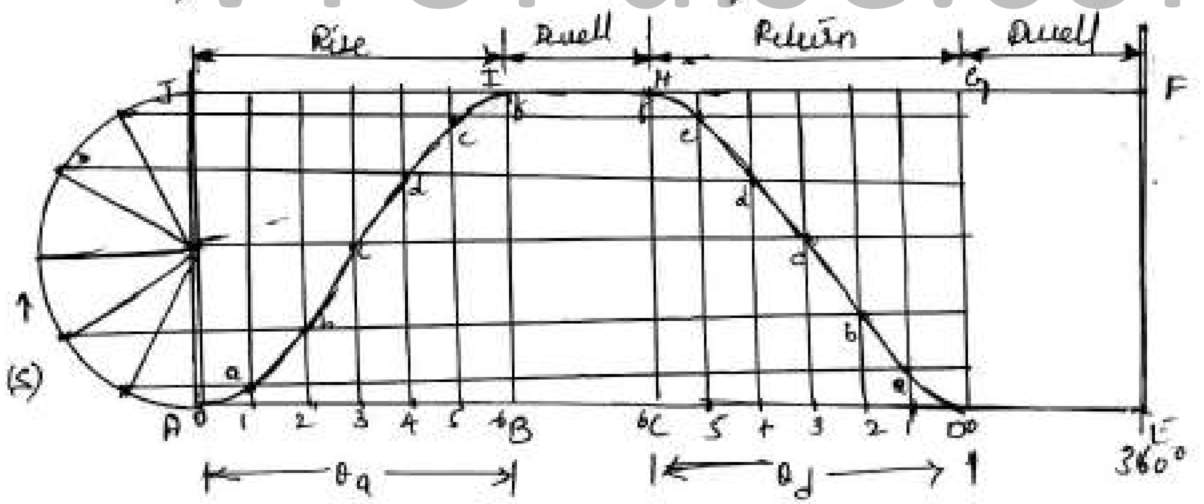
i) On horizontal line Mark AB = θ_a = Angle of rise
BC = Angle of dwell
CD = θ_d = Angle of decent (return)
DE = Angle of dwell. with suitable scale

ii) From 'A' draw ⊥er AJ = S = displacement.
Now complete AEFJ rectangle. IIIly from B, C, D draw ⊥er BI, CH & DG respectively.

iii) Divide AB into even equal parts (say 6 here)

iv) Draw semicircle with "AJ" as diameter & divide semicircle into same equal no of parts.

v) Draw horizontal lines from the divided parts.



Angular displacement ⟶

+ Formulas to calculate Max velocity & Max Acceleration :-

	Max Velocity	Max Acceleration
Out stroke (Rise)	$V_a = \frac{\pi \omega S}{2\theta_a}$	$a_a = \frac{\pi^2 \omega^2 S}{2\theta_a^2}$
Ins troke (Return)	$V_d = \frac{\pi \omega S}{2\theta_d}$	$a_d = \frac{\pi^2 \omega^2 S}{2\theta_d^2}$

(b)

* A cam with 3 cm as Minimum radius is rotating clock wise at a uniform speed of 1200 rpm & has to give the Motion to the knife Edge follower as defined below.

i) Follower to Move outward through 3 cm during 120° of cam rotation with SHM.

ii) Dwell for the next 60°.

iii) Follower to return to its starting position during the next 90° with UARM

iv) Dwell for the remaining period.

Draw the cam profile (a) Follower axis passes through cam axis & (b) Follower axis is offset to the right by 1cm. Also find the Max velocity & acceleration during outward & inward (or) return stroke.

* Displacement diagram:-

The displacement diagram for the following problem is in graph sheet 1.

i) Draw the vertical axis with 1:1 scale & Mark stroke on that axis.

Illy draw the horizontal axis with 1:2 scale & Mark cam displacement on that axis.

ii) Analyze the Motion for Angle of rise & Angle of return & construct accordingly as shown before.

iii) Join all the points got Intersection to draw the graph

iv) Mark 6 division (or) 8 (or) 10 (or) any Even division on the periods to get accurate (or) sharp graph.

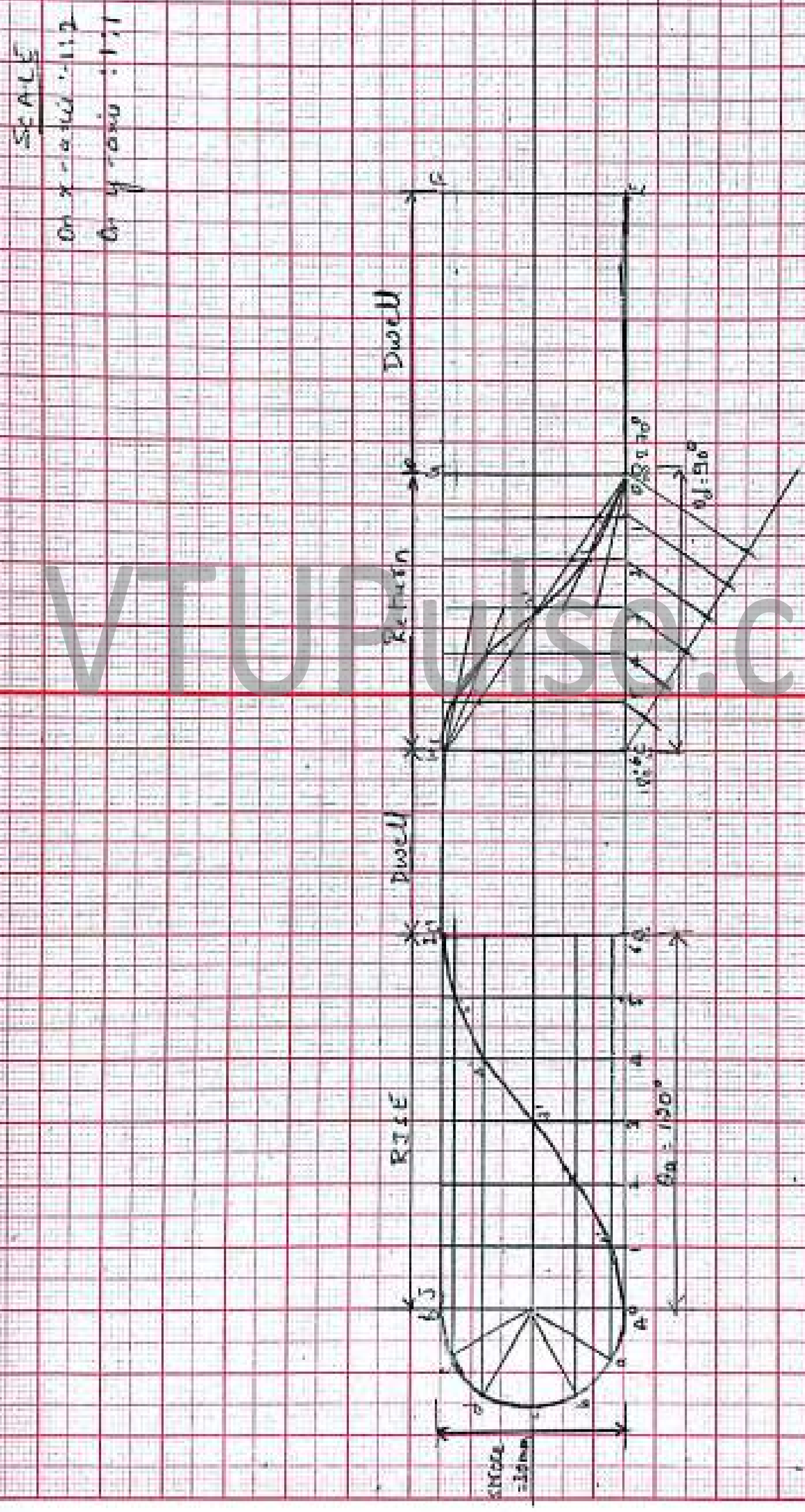
Use 6 division as standard.

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELGAUM. B-09 146264

B.E. / M.B.A. / M.C.A. / M.Tech ________ Semester Examination ________ M M Y Y Y Y

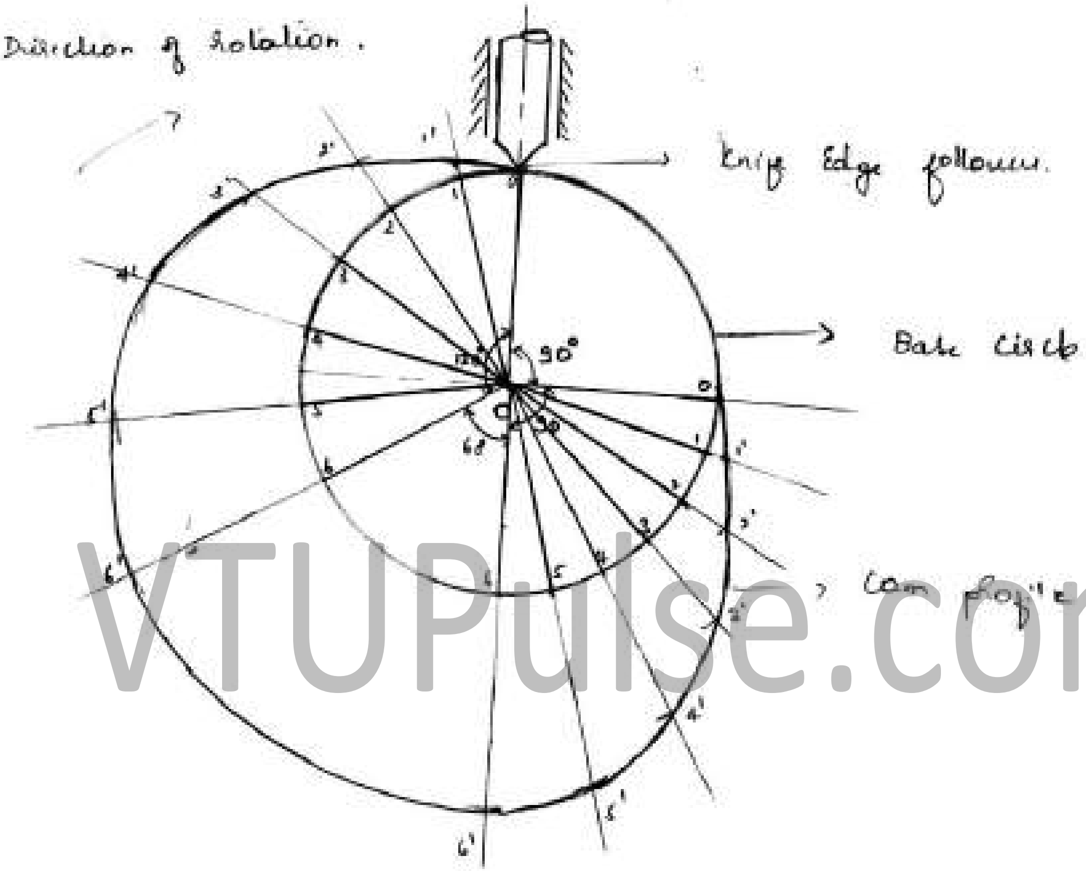
Branch / Title of the Course : ________________________ Subject with code ________________

NOTE : DO NOT WRITE SEAT NUMBER



* Cam problem No1:-

Cam profile for Radial follower:-



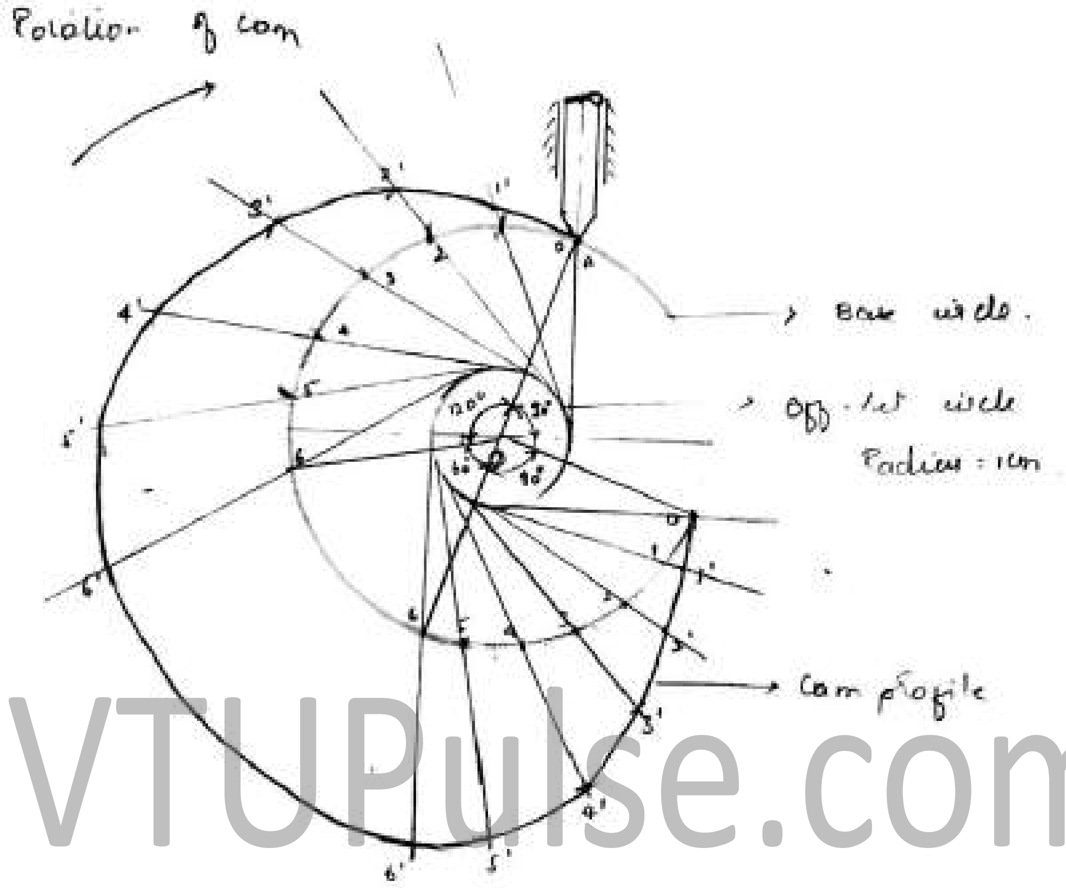
* Construction:-

i) Draw base circle of radius of 30 equal stroke & write Knife Edge follower, so the axis of follower & axis of cam is same.

ii) Mark 120°, 60°, 90°, 90° on the Base cle from anticlockwise direction as Cam is rotating clockwise. Divide this 120° & 90° in to 6 equal parts [$120/6 = 20$, $90/6 = 15°$] & Mark 1, 2, 3, 4, 5, 6.

iii) Measure 1 to 1', 2 to 2', 3 to 3', 4 to 4', 5 to 5' & 6 to 6' & Mark over the Base circle & Joine 1', 2', 3', 4', 5', 6' to get cam profile.

* b) Off-set follower:-



* Construction:-

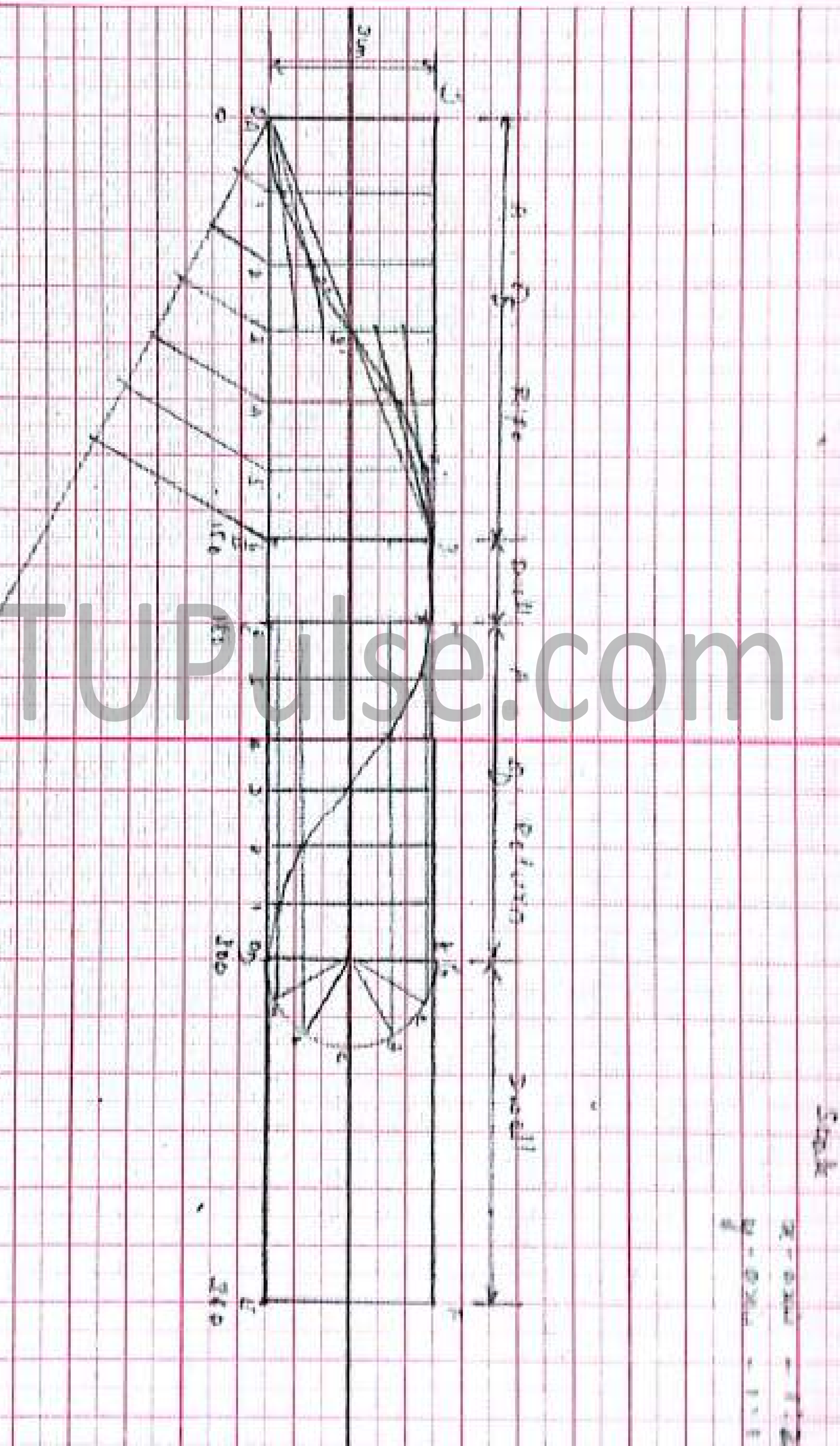
i) Draw base cle of 30 m radius & off-set circle of Radius 1cm. Draw tangent line from off-set cle to base circle.
From the point "A" Join line to "O".
ii) with Reference to OA line Mark 120°, 60°, 90°, 90° & divide 120°, 90° into 6 Equal parts.
iii) Join all the parts divide by drawing tangent line to offset circle.
ii) 11ly Mark 1,1' & 2,2', 3,3', 4 4', 5 5', 6 6' & join all the points to get cam profile.

VISVESVARAYA TECHNOLOGICAL UNIVERSITY, BELGAUM. B-09 145766

B.E. / M.B.A. / M.C.A. / M.Tech ______ Semester Examination

Branch / Title of the Course : ______ Subject with code

NOTE : DO NOT WRITE SEAT NUMBER



③ ⑪

* Calculation:-

i) For outstroke (Rise): (SHM)

we have * velocity $V_a = \frac{\pi \omega S}{2\theta_a}$, $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 1200}{60}$

$\omega = 125.664$ rad/sec

So $V_a = \frac{\pi \times 125.664 \times 3}{2 \times \left(120 \times \frac{\pi}{180}\right)} = 282.74$ cm/sec

(or) $V_a = 2.874$ m/sec.

* Acceleration:- $a_a = \frac{\pi^2 \omega^2 S}{2\theta_a^2} = \frac{\pi^2 \times 125.664^2 \times 3}{2 \times \left(120 \times \frac{\pi}{180}\right)^2}$

$a_a = 53295.6 = 532.95$ m/sec^2

ii) For Instroke (UARM):-

* velocity, $V_d = \frac{2\omega S}{\theta_d} = \frac{2 \times 125.664 \times 3}{90 \times \frac{\pi}{180}} = 480$ cm/sec

$\therefore V_d = 4.8$ m/sec

* Acceleration, $a_d = \frac{4\omega^2 S}{\theta_d^2} = \frac{4 \times (125.664)^2 \times 3}{\left(90 \times \frac{\pi}{180}\right)^2}$

$a_d = 76800$ cm/sec^2 $= 768$ m/sec^2

Example 8.2 :

A cam rotating clockwise at uniform speed of 300 rpm operates a reciprocating follower through a roller 1.5 cm diameter. The follower motion is defined as below.

(i) Outward during 150° with UARM

(ii) Dwell for next 30°

(iii) Return during next 120° with SHM

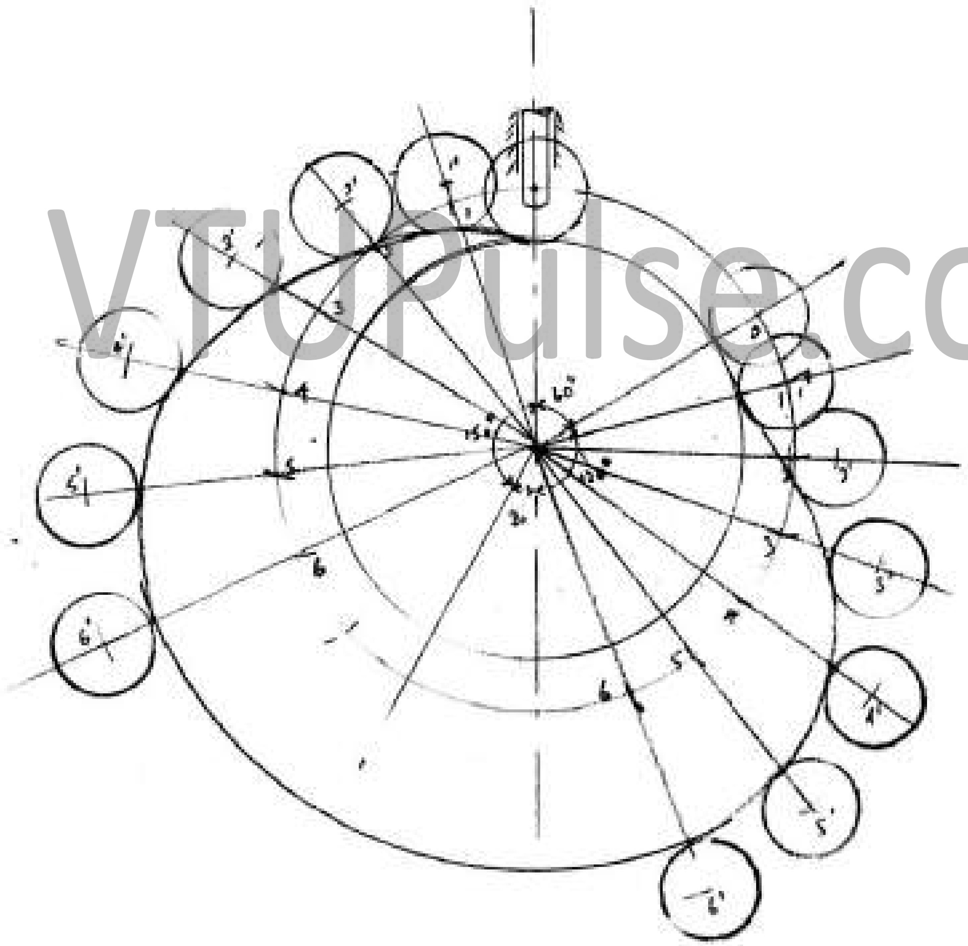
(iv) Dwell for the remaining period.

Stroke of the follower is 3 cm. Minimum radius of the Cam is 3 cm. Draw the Cam profile.

(a) Follower axis passes through cam axis, and

(b) Follower axis is offset to the right by 1 cm.

* Radial cam profile :-



* off-set cam profile :-

Construct off-set circle & draw tangent to that circle & construction is similar as of above profile.

3) solution:-

(11)

Speed = 240 rpm = $\frac{240}{60}$ = 4 rps

The cam makes 4 revolution in on second in 360°

∴ Ascent = 0.05 s = 4 × 360 × 0.05 = 72°

IIly Dwell = 0.0125 s = 4 × 360 × 0.0125 = 18°

Descent = 0.125 s = 4 × 360 × 0.125 = 180°

Remaing period Dwell = 360 − 270 = 90°

Given Acceleration = 3/5 retardation.

* Acceleration :- Increase in rate of change in speed.
* retardation :- decrease in rate of change in speed.

So w.k.T Time of acceleration is reciprocal for retardation time.

So Accleration period = $\frac{5}{3}$ × Deceleration period.

Total period 5+3 = 8

So Accleration period = $\frac{5}{8}$ × 180 = 112.5°

Deceleration period = $\frac{3}{8}$ × 180 = 67.5°

* Return of follower during Acceleration = $\frac{5}{8}$ × 38 = 23.75 mm
* Return of follower during deceleration = $\frac{3}{8}$ × 38 = 14.25 mm

Solution:-

Speed 240 rpm = $\frac{240}{60}$ = 4 rps

The cam makes 4 revolution in 1 second or 360°

Ascent 0.05 s = 4 × 360 × 0.05 = 72°

Ist Dwell = 0.0125 s = 4 × 360 × 0.0125 = 18°

Descent = 0.125 s = 4 × 360 × 0.125 = 180°

Remaining period Dwell = 360 − 270 = 90°

Given Acceleration = 5/3 retardation.

* Acceleration - Increase in rate of change in speed
* Retardation - Decrease in rate of change in speed.

So w.r.t Time of acceleration is reciprocal of retardation time

So Acceleration period = $\frac{5}{3}$ × Deceleration period

Total period = 5 + 3 = 8

So Acceleration period = $\frac{5}{8}$ × 180 = 112.5°

Deceleration period = $\frac{3}{8}$ × 180 = 67.5°

* Stroke of follower during Acceleration = $\frac{5}{8}$ × 38 = 23.75 mm
* Stroke of follower during deceleration = $\frac{3}{8}$ × 38 = 14.25 mm